

Problem Set 9
Due in lecture, Thursday, December 5

1. Consider an asset that has potentially stochastic payoffs in multiple periods, $Z_1, Z_2, \dots$. Suppose that its price is given by:

$$P_t = E_t \left[\sum_{k=1}^{\infty} \frac{1}{(1+\rho)^k} \frac{U'(C_{t+k})}{U'(C_t)} Z_{t+k} \right] \quad \text{for all } t.$$

Show that this implies

$$P_t = E_t \left[\frac{1}{1+\rho} \frac{U'(C_{t+1})}{U'(C_t)} (Z_{t+1} + P_{t+1}) \right].$$

(Note: As always, we assume that C_t is known as of time t .)

2. Consider the model analyzed in the lecture notes on “Forces Pushing Asset Prices toward Fundamentals.” Suppose, however, that the demand of the period 0 noise traders is not fully persistent, so that the market-clearing condition in period 1 is $\rho N_0 + N_1 + S_1 A_1 = 0, \rho < 1$. The market-clearing condition in period 0 remains $N_0 + S_0 A_0 = 0$. How, if at all, does this affect expression (***) for how the noise traders affect the price in period 0? What happens if $\rho = 0$?

3. Consider the model analyzed in the lecture notes on “Forces Pushing Asset Prices toward Fundamentals.” Two features of the various versions of the model that are critical to their tractability are that the shocks are normally distributed and consumption is linear in the shocks. These features cause consumption to be normally distributed, and thus allow us to find agents’ expected utilities. This problem asks you to show that with some natural approaches to modeling performance-based risk, this no longer occurs.

a. Suppose that the representative hedge fund manager, rather than receiving a payment or incurring a cost in period 1, is forced to sell quantity $b(P_1 - E_0[P_1])H, b > 0$, of the risky asset in period 1. Show that in this case, the manager’s consumption is not linear in F_1 .

b. Consider the version of the model without the hedge fund managers, and suppose that A_1 is not exogenous but depends on the success of the period 0 sophisticated investors: $A_1 = \bar{A} + b(P_1 - E_0[P_1])S_0, b > 0$. Show that in this case, the consumption of the sophisticated investors born in period 0 is not linear in F_1 .

(OVER)

4. Consider a setting where lenders are risk neutral and have required rate of return r . Agency costs in investment will result in:

- A. The expected rate of return that lenders receive (net of any agency costs paid by lenders) being less than r .
- B. The expected rate of return that lenders receive (net of any agency costs paid by lenders) being greater than r .
- C. The expected marginal product of capital being less than r .
- D. The expected marginal product of capital being greater than r .

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. Suppose the return on Asset A is riskless and the return on Asset B is risky. Then in equilibrium:

- A. The expected return on Asset B must exceed the expected return on Asset A.
- B. Asset B will provide higher expected return than Asset A to individuals for whom its return covaries positively with their consumption growth, but lower expected return than Asset A to individuals for whom its return covaries negatively with their consumption growth.
- C. Asset B will not be held in equilibrium.
- D. None of the above.

6. (Note: I would not recommend tackling this problem as a way of studying – the algebra involved appears to be a mess. But if anyone tries it and makes some progress on it, I would appreciate your letting me know what you find.) Consider the model analyzed in the lecture notes on “Forces Pushing Asset Prices toward Fundamentals.” In the short-cut approach to performance-based risk, we assume that the amount the hedge fund managers receive in period 1 is a function of $P_1 - E_0[P_1]$. But one can make a reasonable argument that it would be more realistic to model it as a function of $P_1 - P_0$. Specifically, suppose the payout is $aH(P_1 - P_0)$ (where, as before, $a > 0$).

a. With this change in the model, prove (by providing an example) or disprove the following claim: for some parameter values, performance-based risk makes P_0 less responsive to noise traders – that is, that the coefficient multiplying N_0 in the expression analogous to (****) is smaller when $a > 0$ than when $a = 0$. (Hints: (1) I have not solved this problem; I have only gotten far enough to know that the algebra is messy. (2) Assume that each hedge fund manager takes the total demand of hedge fund managers, $A_H H$, as given in choosing his or her own demand.)

b. Explain in words: (1) Why it is reasonable to assume that the payment is a function of $P_1 - P_0$; (2) Why this introduces a force tending to cause performance-based risk to dampen the response of prices to forces pushing them away from their fundamental values.

7. (Note: As with the previous problem, if anyone tries this problem and makes some progress, I would appreciate your letting me know what you find.) Problem 3 above proposes two ways of modeling performance-based risk, but claims that both result in consumption no longer being linear in the shocks. Can you find a way of modeling performance-based risk that is closer than the approaches in the notes to conventional descriptions of performance-based risk (where investors risk loss of funds if the short-term performance of their portfolios is poor) and that preserves linearity? (Hint: I do not have an answer to this question. If I did, I would have used it in the notes and in lecture!)