Economics 280C Problem
Optimal Currency Regime in a Stochastic Keynesian Model

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The model is a stochastic variant on the basic Dornbusch setup, with different assumptions about price behavior and output determination.

The familiar elements are the LM relationship and uncovered interest parity (UIP):

$$m_t - p_t = y_t - \lambda i_t + v_t,$$

$$i_t = i^* + \mathrm{E}_t e_{t+1} - e_t + \varepsilon_t,$$

where v_t and ε_t are, respectively, velocity and risk-premium "shocks" assumed to be i.i.d. such that $\mathrm{E}_t v_{t+1} = \mathrm{E}_t \varepsilon_{t+1} = 0$.

Let w_t be the (log) nominal wage; it is predetermined (and thus sticky on date t), set at date $t-1$ to equal the price level expected for the following period:

$$w_t = \mathrm{E}_{t-1} p_t.$$

Finally, how is output determined? As the intersection of aggregate demand,

$$y_t^{\mathrm{d}} = \delta\,(e_t + p^* - p_t) + g_t,$$

where g_t is an i.i.d. demand shock such that $\mathrm{E}_t g_{t+1} = 0$, and aggregate supply,

$$y_t^{\mathrm{s}} = \theta(p_t - w_t).$$

(a) Under the assumption that m is constant and that the exchange rate therefore *floats*, calculate the equilibrium values of e_t, p_t, and y_t. [Hint: Because all shocks are i.i.d. and mean-zero, the expected value of e and of p is m while that of y is zero.] For simplicity take $i^* = p^* = 0$.

(b) Calculate the variance of output y (where you may assume that the covariance matrix of g, v, and ε is diagonal). (For ease of notation define the composite "financial" shock $\varphi \equiv v - \lambda\varepsilon$.)

(c) Solve the model under a fixed exchange rate, such that e is constant at $\bar{e}$. [Hint: Solve for p and y, noting that $\mathrm{E}_{t-1}p_t = \bar{e}$.]

(d) Compute the variance of output under a fixed exchange rate.

(e) Prove that when the variance of financial shocks σ^2_φ is zero, output variance is lower under a floating exchange rate, and that when the variance of demand shocks σ^2_g is zero, a fixed exchange rate delivers lower output variance.

(f) Now assume that the monetary authority sets interest rates according to a rule

$$i_t = \psi p_t + u_t$$

where u_t is i.i.d. with mean zero. Calculate the equilibrium, including equilibrium output, and the variance of output, σ^2_y, under this type of floating-rate regime. Now the composite financial shock φ, which depends on policy "errors" as well as random investor-preference shifts, is defined as $\varepsilon - u$, and you may assume that it is uncorrelated with g. [Hint: $\mathrm{E}i = i^* = 0$, so $\mathrm{E}p = 0 = \mathrm{E}m$.]

(g) When $\sigma^2_\varphi = 0$, which regime, floating or fixed, delivers lower output variability? What about when financial shocks φ dominate? Why don't the v-shocks from the LM curve above matter anymore?