## Economics 121 PROBLEM SET 3 ANSWER SHEET

### TRUE/FALSE/UNCERTAIN AND EXPLAIN:

1. According to the Stackelberg model, the dominant firm allows the follower to produce a positive amount because it is not possible to completely exclude him from the market.

False/Uncertain: According to the Stackelberg model as presented in class, the follower is already in the market (not an entrant) possibly with sunk fixed costs, so that it may be difficult for the market leader to produce a quantity high enough to cause the follower not to produce. Even in the case where the follower is an entrant not yet in the market, the leader may find it more profitable to accommodate than to exclude (even though exclusion is possible).

2. In a market where firms interact in a repeated context, a monopoly outcome can be sustained as a Nash equilibrium even though many firms operate in the market.

True: See, for example, a cartel. The threat of punishment in the future can deter cheating and keep firms producing their share of the monopoly level.

3. The higher the interest rate, the easier it is for firms in the same industry to sustain a collusive outcome.

False: The higher the interest rate, the harder it is to sustain a collusive outcome. This is because the higher the interest rate is, the more money today is worth compared to future profits. Thus, firms have more of an incentive to cheat today (and make higher profits now even though they will make less profits in the future).

## MULTIPART QUESTIONS

1. In the mainframe computer industry of the 1970s, IBM was the dominant supplier but it did face entry threats. To examine its behavior toward competitors, suppose that IBM produces  $q_1$  and it incurs a cost of  $c_1(q_1) = 6 q_1$  measured in hundreds of thousands of dollars. IBM faces potential entry by Fujitsu, the Japanese mainframe maker. Fujitsu produces a computer that is a perfect substitute for the IBM machine but its production costs are:  $c_2(q_2) = 100 + 12 q_2$  where  $q_2$  is Fujitsu's production level and costs are measured in hundreds of thousands of dollars. Inverse demand for mainframes is given by p(Q) = 120 - Q, where  $Q = q_1 + q_2$  is the total production by IBM and Fujitsu, and again price is measured in hundreds of thousands of dollars. Initially suppose that the incumbent, IBM can credibly commit to a quantity to produce, after which Fujitsu will choose its own quantity.

a. Find Fujitsu's reaction function.

 $\mathbf{p}_2$ 

Fujistsu's profit function is:

 $= Pq_2 - C(q_2)$ = (P(q\_1,q\_2) - mc\_2)q\_2 - F = (120 - q\_1 - q\_2 - 12)q\_2 - 100

Maximizing with respect to  $q_2$  (note that the fix cost drop out) gives  $r_2(q_1) = \frac{1}{2}(108 - q_1) = 54 - \frac{1}{2}q_1$ 

b. If IBM accommodates entry, find IBM's profit-maximizing quantity and its resulting profits. If IBM accommodates entry, it will then act as a Stackelberg leader. It gets to choose its quantity first knowing what Futjisu will produce in response. Taking that into account, IBM's

profit function will be:

 $\begin{array}{rl} p_1 &= (120 - q_1 - q_2 - 6)q_1 \\ &= [120 - q_1 - (54 - \frac{1}{2} q_1)]q_1 \\ &= (66 - \frac{1}{2} q_1)q_1 \end{array}$ Max profits w.r.t. q<sub>1</sub>,  $\begin{array}{r} q_1^* &= 66 \\ \text{Under this scenario, Fujitsu's best response would be} \\ r_2(q_1 = 66) = 54 - 66/2 = 54 - 33 = 21 \end{array}$ Total quantity is:  $\begin{array}{r} q_1 + q_2 = 87 \\ \text{Market price is:} & 120 - 87 = 33 \\ \text{Profits for IBM:} & (P - c)q_1 = (33 - 6)66 = 30*66 = 1980 \end{array}$ 

c. Alternatively, IBM can attempt to deter entry by Fujitsu by engaging in "limit pricing." In fact, it would set a quantity so that Fujitsu would not be able to make a profit. This is not a question.

*d.* If IBM produces to limit Fujitsu's entry, verify that  $q_1 = 88$  is the quantity that results in the limit price, and find that price and IBM's associated profit.

- While it is sufficient to show that at  $q_1 = 88$ , Fujitsu will make zero profit, let's derive the answer. IBM knows that Fujitsu's profit function is

$$=(120-q_1-q_2-12)q_2-100$$

 $\mathbf{p}_2$ 

And that its best response is  $r_2(q_1) = (54 - \frac{1}{2} q_1)$ . Mathematically, this means that we can express Fujitsu's profit solely as a function of IBM's quantity. Strategically, this means that IBM can choose a quantity such that Fujitsu's best response is not to enter (recall the best response function only takes into account a <u>marginal</u> considerations: it takes into account marginal costs but not fixed costs. If IBM limit prices it will pick the quantity such that Fujitsu's profits are zero:

$$\begin{array}{ll} p_2 &= (120-q_1-(54-\frac{1}{2}\,q_1)-12)(54-\frac{1}{2}q_1)-100=0 \\ 0 &= (66-\frac{1}{2}q_1-12)(54-\frac{1}{2}q_1)-100 \\ 100 &= (54-\frac{1}{2}q_1)^2 \\ 10 &= 54-\frac{1}{2}q_1 \\ q_1 &= 88 \end{array}$$

Check: at  $q_1 = 88$ , Fujitsu produces 54 - 44 = 10, market price is 120 - 88 - 10 = 22 and profits are (22 - 12)10 - 100 = 0.

#### e. Will IBM prefer to deter entry or accommodate entry? Prove it.

First note that if IBM succeeds in excluding Fujitsu, the total quantity produced will be 88 and the market price 32. Thus IBM's profits will be,

$$\begin{array}{ll} p_2 & = (P-c)q_1 \\ & = (32-6)88 \\ & = 2288 \end{array}$$

Since this yields higher profits than accommodating, IBM will choose the limit pricing strategy.

2. Practice Problem 6.4 in the PRN text (pages 313-314). This problem is good practice and we strongly recommend you work through it without the solutions.

Now consider how the Dixit model and how it applies to the "real world," which in this case means the CSG game. Refer to the market profiles under the CSG section of the course web page.

f. In the Dixit model, an investment in a capacity q<sup>k</sup> lowers marginal costs for all units up to that capacity choice. Rank the markets in terms of how much ex-post marginal costs compare to ex-ante marginal costs, which will include your capacity costs. (Hint: in calculating your ex-ante marginal costs, think about the per-period cost of capacity).
Ex-ante MC: Ignoring the interest rate, your ex-ante marginal costs will be marginal costs plus the per-period costs of capacity. So, for market A the average capacity costs is \$800 and last for 4 periods, giving \$200 per-period costs.<sup>1</sup> Marginal costs are \$50. Thus, in any one period, 1 unit of production would costs \$250.

**Ex-post Marginal Costs:** For now we will ignore depreciation. As we will see, depreciation will be an important factor in determining ex-post marginal costs. If capacity were completely sunk (depreciation in the first period of use were 100%), then the ex-post marginal costs would just be the marginal costs.

	Ex-ante Marginal	Ex-post Marginal	Ratio
	Costs	Costs	
Market A	800/2+50 = \$250	50	5
Market B	50/10 + 222 = 227	222	1.02
Market C	500/3+20 = 167+20 =	20	9.35
	187		
Market D	4000/6 + 200 = 666 +	200	4.33
	200 = 866		

Recall now that, in the Dixit model, the upfront sunk investment makes firms stronger by lower its effective ex-post marginal costs. Again, ignoring depreciation, this difference is strongest in market C.

g. How does the "elasticity of marginal costs for output greater than capacity" in each of the CSG market (again, look in the market profiles) compare to the assumptions for production past capacity in the Dixit model? Very generally, how do you think this will change the results of the Dixit model?

In the Dixit model, an investment in capacity  $q^k$  gives you ex-post marginal costs of *w* if you produce below  $q^k$ . Production above that amount and marginal costs increase to w+r. In the CSG, producing past capacity does not bring about a sharp increase in your marginal costs, it increases slowly (and at a linear rate). In general, if one firm (the "incumbent" in the Dixit model) has built up a larger amount of capacity, it will give them lower marginal costs over a larger range of output. If the elasticity of MC is low, then producing past capacity is not very costly. But this also applies to the "follower." The key, therefore, is how it changes exante costs and ex-post costs. The higher the elasticity, the more the two costs will diverge.

*h.* One key feature of Dixit model is that investment in capacity is completely sunk. How does this assumption hold up in each of the markets (hint: consider depreciation).
On the one hand, consider what happens when depreciation is high. That is, once you make a capacity decision, the costs are sunk, you cannot get it back. This is similar to the Dixit model. This is reflected in market D, where, once you invest in a certain capacity, the most you can get back is 50% of its value.

If, however, depreciation were low, in any given period, you have the option of using that unit of capacity or selling it. Consider market B. There, you only loose 10% of its value. If you over invest in capacity, you can still sell it off and get most of its value back. The commitment value is not, therefore, very high.

<sup>&</sup>lt;sup>1</sup> To see this more clearly, consider what would happen if you built 1 unit of capacity. Suppose that in each period, you only produced 1 unit. At the end of 4 periods, you will have produced 4 units. Total production costs would be \$800 (capacity of 1 unit) plus \$50 MC for each period = \$1000. The average MC for any 1 unit will thus be \$250.

- *i.* In the Dixit model an incumbent may choose a level of capacity that deters entry. Describe how the entry fee in the different markets affect the limit quantity. The greater the entry costs, the easier it will be for an incumbent to deter entry. In market D, for example, potential entrants need to know if they will be able to cover the high entry costs. If they observe one firm with a large capacity level, to the extent that capacity is sunk (see part c), the larger the entry fee easier it is to deter entry.<sup>2</sup>
- *The Dixit model assumes all goods are homogeneous products. Describe how product differentiation as modeled in the CSG game might change the Dixit model.* Product differentiation in the CSG means that firms are relatively isolated from competition. In market B, one firm can increase its capacity and output substantially Increased capacity, therefore, is unlikely to deter entry. Market D, however, firms compete on price (note that this is even a stronger form of competition than the Dixit model assumes). A capacity in that market, therefore, may be a significant threat.
- k. Consider the entry decisions of the 8 teams in period 1. Given random costs, what kind of decision rule might a firm follow in deciding which of the market to enter? Given this decision rule, which firm(s) will be the "first mover" in each market.
  Many teams decide to enter a market in which they expect to have lower than average marginal costs. Such a rule would mean that the "first mover" in a market has lower costs. Note that, in a Dixit model context, this gives the low-cost firm a double advantage.
- 1. Which of the markets would you expect the Dixit model to apply to? In most of the cases above, Market D fits a market in which commitment value of capacity is important. The Dixit model is least relevant to Market B, with its relatively isolated markets in which entry deterrence does not seem credible.
- 3. An industry composed of eleven (11) identical firms has decided to form a cartel. Initially, all firms join the cartel. Demand for the product is: D(p) = 100 p. Each firm has total cost function:  $C(q) = 4q + q^2$ . Firms decide on the quantity to produce, and the market price adjusts to clear the market for the homogeneous product.

*a. Compute the cartel solution for price and the resulting profit of an individual firm.* From the notes on the "Cartel Solution" we know:

$$MR(Q) = 100 - 2Q = 4 + 2q_i = MC_i(q_i)$$

Since all firms are identical Q = 11q where q is how much each firm produces so

$$100 - 22q = 4 + 2q \implies 24q = 96 \implies q^{C} = 4, \ Q^{C} = 11q^{C} = 44, \ p^{C} = 100 - 44 = 56$$
$$\boldsymbol{p}^{C} = \boldsymbol{p}^{C}q^{C} - \boldsymbol{C}(q^{C}) = 56 \cdot 4 - 32 = 192$$

The firms attempt to achieve the cartel solution for current period. One firm, however, considers "chiseling" on its cartel quantity by producing its profit-maximizing level, assuming that the remaining ten (10) members <u>will produce so as to maintain the cartel price</u>.

b. Find the optimal quantity the "chiseler" will produce and how much profit it makes from chiseling.

The chiseler takes the output of others as given, so  $q_i = 4$ ,  $j \neq i$  and so his profit is given by

 $<sup>^{2}</sup>$  You need to, of course, consider how much money you make in the market. If you were guaranteed to make \$100,000 per period in Market D, the high entry fee is not much of a deterrent.

$$\left(100 - \sum_{j=1}^{11} q_j\right) q_i - 4q_i - q_i^2 = \left(100 - \sum_{j \neq i} q_j\right) q_i - q_i^2 - 4q_i - q_i^2$$
$$= (100 - 40) q_i - 4q_i - 5q_i^2 = 56q_i - 2q_i^2$$

Taking the FOC

$$56 - 4q_i = 0 \Rightarrow q_i = q^D = 14, Q^D = 14 + 40 = 54, p^D = 46, p^D = 46 \times 14 - 200 = 392 > 192$$

Suppose that all firms expect that after ten (10) years a new product will become available that will make their product obsolete. In the meantime, they attempt to abide by the annual cartel production levels for each of the ten years.

## c. If the relationship is repeated for each of 10 years, will the firms be able to achieve the cartel outcome? Explain why or why not.

The unique Nash Equilibrium in a Cournot Oligopoly is for everyone to produce the Cournot quantity where each firm maximizes it's own profit taking other's actions as given. Since the time horizon is finite, Selten's theorem implies that the only Subgame Perfect Nash Equilibrium (SPNE)the firms can play is the Nash Equilibrium over and over. The Cartel strategy is not a Nash equilibrium strategy in a one shot game and therefore cannot be maintained as a SPNE.

Aside: Solving for the Cournot quantity. Firm *i* maximizes its profits

$$\max_{q_i} \left( 100 - \sum_{j \neq i} q_j \right) q_i - q_i^2 - 4q_i - q_i^2 = \max_{q_i} \left( 96 - \sum_{j \neq i} q_j \right) q_i - 2q_i^2$$

so the FOC implies that  $96 - \sum_{j \neq i} q_j - 4q = 0$  and by symmetry  $q_i = q_j = q$ ,  $\forall i, j$ 

$$96 - 10q - 4q = 0 \implies 14q = 96 \implies q^{N} = 96/14 = 48/7 \cong 6.86, Q^{N} = 11 \times 48/7 = 528/7 = 75.43$$
$$p^{N} = 100 - 528/7 = 172/7 = 24.57,$$
$$p^{N} = p^{N}q^{N} - C(q^{N}) \cong 6.86 \times 24.57 - 4 \times 6.86 - (6.86)^{2} = 168.55 - 74.50 = 94.05 < 192$$

Now suppose that it is discovered that a superior product will <u>never</u> be invented so that the firms can continue to supply the market forever.

# *d. Can the cartel outcome be sustained in this case? How might the firms attempt to achieve this? Identify key factors that will facilitate collusion in this case.*

By the Folk Theorem, the Cartel solution may be maintained in this case for a high enough discount rate d < 1. One way would be to use a Trigger Strategy, so that all firms always produce the cartel quantity unless anyone deviates in which case all of the firms produce the Nash quantity.

Collusion requires effective detection mechanisms to observe chiseling as well as barriers to prevent entry.

<u>Aside:</u> We can actually solve for the required discount factor for a trigger strategy to work. According to the notes this requires

$$d > \frac{p^{D} - p^{C}}{p^{D} - p^{N}} \approx \frac{392 - 192}{392 - 94.05} = \frac{200}{297.95} \approx .67$$