



# Monopoly Pricing With Network Externalities\*

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## Abstract

How should a monopolist price a durable good or a new technology that is subject to network externalities? In particular, should the monopolist set a low “introductory price” to attract a “critical mass” of adopters? In this paper, we provide intuition as to when and why introductory pricing might occur in the presence of network externalities. Incomplete information about demand or asymmetric information about costs is necessary for introductory pricing to occur in equilibrium when consumers are small.

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# 1 Introduction

How should a monopolist price a durable good or a new technology that is subject to network externalities? Should the monopolist use declining prices to skim off consumer surplus, or, alternatively, should it launch the product with low “introductory” prices to attract a “critical mass” of adopters? These important questions have received surprisingly little attention in the particular context of network externalities.

In this paper, we characterize the equilibrium price path when a monopolist sells a durable good that confers a network externality on a collection of rational buyers. Our principal goal is to establish whether equilibrium prices can increase over time under various assumptions about size of consumers and demand and cost information. We find that, when there are network externalities, under a variety of circumstances, prices increase over time. We provide intuition as to when and why introductory pricing can occur in the presence of network externalities.

Early development of telephone service supplies a near-perfect example of a monopoly over a service having network externalities. A user derives value from a communications network in rough proportion to the total number of subscribers. The telephone system in the U.S. was a monopoly based on Bell’s 1876 patents over the basic technology. Average monthly fees charged by the unregulated telephone companies rose steadily in the early 1880s, nearly doubling over a four-year period. Thereafter the price path flattened, only to plummet when the patents expired in 1893.<sup>1</sup>

On-line information services offer a more recent illustration of introductory pricing. First *CompuServe*, and later *Prodigy*, were introduced with a small sign-up charge and a low monthly fee. As the customer base grew, the services raised prices gradually.

Users need not be connected by a physical network to realize network externalities. Users of computer operating systems and some general-purpose application packages receive an indirect externality as complementary hardware and software products become available. Computer vendors adopt marketing practices designed to take advantage of this spillover. Makers of new hardware platforms are known to offer especially attractive licensing terms to early developers of compatible software. Introductory pricing is also a common strategy when launching new software operating systems

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<sup>1</sup>Detailed supporting data for this example and others that follow are available from the authors upon request.

and other general purpose software for which network externalities are important: specifically, it works to enhance the product's quality by attracting "lead users" (mainly software developers).<sup>2</sup>

These cases establish that introductory pricing may be an equilibrium pricing strategy when network externalities prevail and monopoly power is present. Our goal is precisely to describe situations in which introductory pricing is an equilibrium outcome. Furthermore, in each model we present, introductory pricing fails to occur *unless network externalities are present*. In this way we have been able to isolate the role played by network externalities in introductory pricing.

In each of the various cases we study, the good has constant quality though its value will increase as more consumers purchase it due to network externalities. Purchases can take place in one of two time periods and repeat purchases are not permitted, nor are there resale markets.

Our paper draws on three lines of previous research. The first is the growing literature on the adoption of innovations with network externalities. Farrell and Saloner (1985), Arthur (1989), and others examine equilibrium adoption of "unsponsored" (or nonproprietary) innovations, ignoring the issue of pricing. Katz and Shapiro (1985, 1986), on the other hand, consider the pricing of competing "sponsored" (or proprietary) innovations. They find introductory pricing in equilibrium, but these low first-period prices are caused by a duopolist's rush to establish an installed base ahead of its rival.<sup>3</sup>

The second line of research is the vast literature that endeavors to verify the so-called "Coase conjecture" regarding a monopolist selling a durable good.<sup>4</sup> Coase (1972) claimed that the price set by a monopolist who is unable to commit to future prices will quickly converge to marginal cost as the time between sales becomes arbitrarily short. The Coase conjecture was confirmed and disconfirmed under a variety of conditions.

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<sup>2</sup>When first available, Microsoft licensed MS-DOS to Original Equipment Manufacturers for a flat fee, and for a limited time that fee was reduced by half. See Manes and Andrews (1993). Further, new versions of applications packages are often introduced at low prices for a limited time, after which the price jumps dramatically. See *Business Week*, November, 1993, pp. 86-8 for several examples.

<sup>3</sup>Gallini and Karp (1989) also find introductory pricing when there is consumer lock-in, due to firm (or product) specific investments, and repeat purchases.

<sup>4</sup>Stokey (1981); Bulow (1982); Sobel and Takahashi (1983); Fudenberg, Levine and Tirole (1985); Ausubel and Deneckere (1986); Gul, Sonnenschein and Wilson (1986); Kahn (1986); Bagnoli, Salant and Swierzbinski (1989); Von der Fehr and Kühn (1991); Kühn and Padilla (1992).

In all cases the equilibrium solutions obey what Hart and Tirole (1988) call “Coasian dynamics.” Coasian dynamics consist of two properties: (i) higher valuation adopters make their purchase no later than lower valuation adopters (the *skimming property*) and (ii) equilibrium price is nonincreasing over time (the *price monotonicity property*). In this paper, we show that the second property need not always hold when network externalities are present.

Finally, the Marketing Science literature has examined pricing with “experience” or “network” effects.<sup>5</sup> These papers typically assume that buyers obey some rule that is not necessarily rational, or alternatively that they are imperfectly informed about the existence or the quality of the good. In the latter case, buyers learn over time either by repeat purchases or through word of mouth.<sup>6</sup> By contrast, we assume that buyers are perfectly rational agents.

We begin our analysis with the case of perfect information in Section 2. We show that, if each buyer is “small,” then discounted prices *must* decrease over time. In other words, Coasian dynamics prevail. If, instead, consumers are “large,” we can construct examples in which *discounted* prices rise over time by carefully selecting from among multiple equilibria.

In Section 3, we assume imperfect information about consumers’ valuations. Again we treat the cases of small and large buyers separately. In both instances, we find equilibria in which prices increase over time. However, the intuition for the result differs between the two cases. When buyers are small, the inducement of a low first-period price is needed to compensate for the uncertainty of an early adoption. In contrast, when buyers are large, delaying a purchase can actually increase the probability that other buyers will eventually adopt. In this latter case the firm sets a lower first-period price to counteract consumers’ tendency to delay adoption.

Finally, in Section 4 we let the firm’s cost be unknown to buyers. Again, we find perfect Bayesian equilibria in which discounted prices rise over time. In this case, introductory prices serve as a signal of low cost, thus raising early buyers’ expectations about the likelihood of future sales. The lower the seller’s cost is, the greater future sales will be, and thus the higher the expected utility of a purchase today is.

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<sup>5</sup>Bass (1980), Clarke, Darrough and Heineke (1982), Kalish (1983), Besanko and Winston (1990).

<sup>6</sup>On this see Vettas (1993).

## 2 The Certainty Case

We begin by assuming that there is perfect information about demand, cost, and the quality of the product. However, we assume that the monopolist is unable to set prices based on buyers' types, either because it cannot observe some individual characteristic, or because it is precluded from price discrimination.

Each buyer's per-period valuation of the good depends on her type as well as on the number of other buyers who have purchased the good—the essence of a network externality. This is represented as  $u_i(n_t)$ , the utility derived by consumer  $i$  given the cumulative number of purchases through period  $t$ ,  $n_t$ . Each adopter demands at most one unit. Since there is no possibility of resale, a buyer will make a purchase in any given period only if she has not done so earlier. Once purchased, the good provides a stream of benefits that each consumer discounts according to the discount factor  $\delta$ . Finally, we deduct the current price,  $p_t$ , to arrive at the net payoff.

In each period, the firm first quotes a price; then, all consumers who have still not made a purchase simultaneously decide whether to purchase in that period, given information about the current price and all previous purchase decisions.

We say that a buyer is “small” when her decision to purchase the good has no effect on the payoff to other buyers or on the strategies they choose and on the monopolist's prices. This would be true if there were a countably or uncountably infinite number of them. A buyer is “large” if her purchase decision has a noticeable effect on other buyers' payoffs and decisions. We find that the occurrence of introductory pricing depends on the “size” of buyers.

**Proposition 1** *If all buyers are small, then in a subgame-perfect Nash equilibrium discounted prices cannot rise between periods in which sales occur.*

The proof of this and other propositions in the paper may be found in the Appendix. Intuitively, if discounted prices rise, each consumer can cut her outlay by advancing the purchase, and also gains utility from consumption during the interim period. Since no consumer perceives its decision to make an earlier purchase will have an effect on other consumers' decisions or on the price path, they will proceed with the purchase.<sup>7</sup>

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<sup>7</sup>This is the intuition behind Stokey's (1979) results in the absence of network externalities. She finds that, when the firm is unable to commit to a price path, all sales will occur in the first period.

Note that the result does not depend on the strength of the network externality. Discounted prices cannot rise even in complete absence of network effects.<sup>8</sup> The result does depend, however, on the assumption that buyers are small. With large buyers, examples can be found whereby an increasing discounted-price sequence arises in equilibrium.<sup>9</sup> These examples exploit the multiplicity of equilibria under network externalities: buyers coordinate their purchases so as to “punish” any deviations from a proposed equilibrium with increasing prices. In this context, a late buyer may not want to advance her purchase decision under the fear that this will trigger a reversion to a “bad” equilibrium.<sup>10</sup> Arguably, the equilibrium behavior that supports this type of equilibria is not very realistic. This reinforces the main idea of this section, namely that, without some form of uncertainty, it is unlikely that prices will increase in equilibrium, i.e., price skimming dominates introductory pricing.

In the next sections we explore how uncertainty about demand or cost can result in introductory pricing. Once again, we consider separately the cases of “large” and “small” buyers, but now increasing prices are possible in both cases.

### 3 Incomplete information about demand

#### 3.1 Strategic buyers

Suppose there are two potential buyers and two periods. The  $i$ -th buyer’s utility is given by  $v_i$  if she is the only one who buys, and  $v_i + u$  if both buyers make a purchase. Here  $u$  measures the network externality; it is the same for both buyers and its value is common knowledge to the seller as well as to the buyers.  $v_i$  measures the “standalone” utility; its value is each buyer’s private information and is independently distributed from the other buyer’s  $v_j$ . The prior distribution of  $v_i$  is uniform on the interval  $[0, 1]$ . Production cost is assumed to be zero.

This setup is similar to the model of incomplete information presented in Farrell and Saloner (1985). The principal distinction is that we consider

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<sup>8</sup>Nominal price can nevertheless rise where the rate of increase depends on the discount factor and on the strength of the network externality.

<sup>9</sup>One such example is available from the authors upon request.

<sup>10</sup>More than introductory pricing is possible when buyers are large. In fact, there may exist a continuum of equilibria. Equilibrium refinements such as coalition proofness or risk dominance, however, may drastically reduce the set of equilibria.

a proprietary innovation which is therefore priced. Farrell and Saloner consider an “unsponsored” innovation, thus concentrating on issues of buyer coordination.<sup>11</sup>

We focus on interior equilibria in which, with probability strictly between 0 and 1, a sale is made in each period (provided there is unsatisfied demand). As we will see, to ensure the solution is interior,  $\delta$  and  $u$  cannot be too large. If  $\delta$  is close to 1, then only a corner solution exists in which no sales occur in the first period. In this extreme case it makes no sense to talk about the evolution of equilibrium prices since no sales are made at the initial price. If, on the other hand,  $u$  is very large, then the network externality swamps the uncertainty and the standalone value, and so all consumers who buy will buy early on. Again, the offer of higher second-period prices is not exercised. For an open set of values of  $\delta$  and  $u$ , however, we can show that a unique Perfect Bayesian Equilibrium exists that displays introductory pricing with certainty.

**Proposition 2** *If  $0 < u < 1/2$  and  $\delta$  is close to (but lower than)  $\bar{\delta}(u)$ , where*

$$\bar{\delta}(u) \equiv \frac{4(1-u)^2}{4(1-u)^2 + 2u^3(1-u) - u^4}, \quad (1)$$

*then there exists a unique, interior Perfect Bayesian Equilibrium in which second-period discounted price exceeds first-period price with probability 1.*

The proof of the result is in the Appendix. The intuition is as follows. Consider first the case when  $\delta \sim 1$  and  $u = 0$ . It is well known from the literature on bargaining and durable goods pricing that a monopoly seller will not price discriminate over time: discounted prices are nearly constant over time and almost no sales occur in the first period. Accordingly, buyers will choose to wait to purchase since utility is not discounted and they retain the option of a better outcome.

Now suppose that  $u > 0$ . Network externalities introduce a new factor into the buyers’ decision besides the time profile of prices. A buyer must weigh the impact of her decision on the likelihood that the other buyer will purchase. As shown in the proof, by foregoing a purchase in the first period, the likelihood of a sale in the second period to the other buyer actually *increases*. The reason is that, *if no sales occur in the first period, then both buyers’ combined willingness to pay is smaller since neither one is guaranteed*

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<sup>11</sup>Also, Farrell and Saloner’s payoffs have a more general specification of network externalities.



*the network externality if she buys.* The seller responds with a price much lower than if no sale occurred in the first period, thereby encouraging a second-period purchase by either buyer.

In contrast to buyers, the seller prefers to make all sales in the first period. Although delaying the purchase increases expected network size, it also decreases expected profits. Therefore, the seller has an incentive to lower first-period prices so as to discourage buyers from delaying first-period purchases in an attempt to achieve low second-period prices.

When the discount factor is sufficiently small, equilibrium price decreases with probability one. Since the future is heavily discounted, the only difference between periods is that in the second period, with positive probability, the monopolist will have lowered its priors with respect to the buyers' valuations, which in turn leads to lower prices. So, for  $u > 0$  and  $\delta \sim \delta(u)$  equilibrium prices increase over time, whereas for  $\delta \sim 0$  equilibrium prices decrease over time. It follows by continuity that for intermediate values of  $\delta$  and  $u$ , equilibrium price increases or decreases with positive probability.

There is an interesting parallel between the equilibrium just described and the one found in Farrell and Saloner's (1985) adoption game with network externalities. In both cases, medium-valuation adopters play "bandwagon" strategies: to adopt (or buy) in the second period if and only if an adoption (purchase) was made in the first period.

Finally, it can also be shown that, in equilibrium, whatever the magnitudes of the network externality and the discount factor, the equilibrium is inefficient: welfare-increasing adoptions (purchases) are delayed, or, in some cases, never made. This is not surprising in light of Farrell and Saloner's "excess inertia" result for unsponsored innovations. When the firm prices a proprietary innovation (product), the equilibrium can very well be less efficient—and it is.

### 3.2 Demand Uncertainty and Lead Users

We now return to the assumption of "small" buyers, assuming there is a continuum of buyers who can purchase in one of two periods. Each buyer can be one of two types:  $H$  or  $L$ . A crucial assumption is that only  $H$ -type buyers confer network benefits on other buyers. Accordingly, we can treat these buyers as "lead users," to borrow a term from the marketing literature. These users contribute in a decisive way to the amount of complementary products and services that generate network benefits. For example, if the basic product was a new software operating system, then the lead users

might be developers of software applications.

The utility of  $H$ -type buyers is given by  $u^H = v + x$ , where  $x$  is the number of  $H$ -type buyers that eventually buy the product.  $L$ -type buyers receive utility  $u^L = x$ . For simplicity, we assume there is no discounting or interim utility, so all that matters to buyers is the eventual number of  $H$ -type adopters and the price paid.

The measure of potential  $H$ -type adopters is given by  $\alpha$ . The value of  $\alpha$  is uncertain to both buyers and seller. It can take on the value  $\bar{\alpha}$  or  $\underline{\alpha}$ . Both buyers and the seller hold a common prior probability  $\rho$  that  $\alpha = \bar{\alpha}$ . Finally, the seller has a constant marginal cost  $c$ .

**Proposition 3** *Suppose that*

$$\underline{\alpha} < c < \bar{\alpha} < \underline{\alpha} + v \tag{2}$$

$$\bar{\alpha}v < (1 - \bar{\alpha})(\bar{\alpha} - c). \tag{3}$$

*If  $\rho$  is sufficiently low, then there exists a unique Perfect Bayesian Equilibrium in which expected second-period price is higher than the first-period price.*

The proof can be found in the Appendix. It is worth noting that the set of parameter values determined by (2)–(3) is non-empty. For instance, they are satisfied by  $\underline{\alpha} = 0$ ,  $c = .1$ ,  $\bar{\alpha} = .2$ , and  $v = .3$ .

The intuition for this result can be seen in the following way. The new product can either be a success or a failure (“good” or “bad”), corresponding to whether the likelihood of “lead users” is large or small. In equilibrium, this is known at the beginning of period 2. If the product is “good”, then the seller prices low in order to attract the largest fraction of buyers, namely the followers. If, on the contrary, the product is “bad”, then the seller sets a high price, knowing that she will only sell to high-valuation consumers. Now, high-valuation consumers are more optimistic than the seller about the possibility that the product is “good” and hence that a relatively low price will be set in period 2. Therefore, for sales to occur in the first period rather than the second, the seller has to set a price that is lower than the expected second-period price.

Condition (2) and the assumption that  $\rho$  is low guarantee that it is worth attracting low valuation users if and only if the fraction of lead users is high. Condition (2) also guarantees that second period price is higher when the fraction of lead users is lower. Finally, condition (3) implies that it is optimal to price in such a way that consumers separate rather than pool.

## 4 Asymmetric information about cost

Our last explanation for introductory pricing hinges on asymmetric information about production costs. Specifically, we suppose consumers are not perfectly informed about the seller's unit cost. Since we want to concentrate on the effects of asymmetric information, we avoid the issue of consumer's timing of purchases by assuming that the monopolist is selling to two consumers who arrive sequentially.

Each consumer can be one of two types: high valuation (type  $h$ ) or low valuation (type  $l$ ) with probability  $\alpha$  and  $1-\alpha$ , respectively. A high valuation consumer has a utility of  $v$  if she is the only one to buy the product, and  $v+u$  if both consumers buy the product. A low-valuation consumer receives 0 utility if she is the only one to buy the product and  $u$  if both consumers buy. Utilities are realized after both adoption decisions have been made.<sup>12</sup>

Finally, the seller can be of two types, high cost ( $c$ , type  $H$ ) and low cost (zero, type  $L$ ). The seller's cost is unknown to buyers at the time of purchase.<sup>13</sup> In this event we once again find an equilibrium in which the seller sets an increasing price sequence.

The formal statement of this result and its proof may be obtained from the authors upon request. The proof is by construction, namely by construction of a separating equilibrium. We derive conditions such that, at equilibrium and after a sale in period 1, the monopolist optimally sells to both types of buyer in period 2 if it has low cost, but only to the high-valuation buyer if it has high cost.

Knowing this, and given that there are network externalities, a buyer's expected utility (and willingness to pay) in the first period is higher the more she believes the seller to be a low-cost firm. This, in turn, creates an incentive for high-cost sellers to masquerade as low-cost sellers in the first period. Finally, to distinguish itself from a high-cost seller, the low-cost seller has to set a very low price in period 1.

We should note that this equilibrium is very similar to the one found in Bagwell (1989). In his model, consumers must decide whether, after visiting the seller in the first period, to incur a fixed cost to return in the second period. Bagwell finds that a seller will employ introductory pricing to signal low cost. First-period consumers return to the seller when they

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<sup>12</sup>Again, this simplification is made with the sole purpose of isolating the effect of asymmetric information about cost.

<sup>13</sup>Since we consider equilibria with separation, the probability that the seller's cost is low is irrelevant so long as it is positive.

believe the seller is low cost, and hence, will charge a low second-period price. In our model first-period consumers care about future prices because the probability of future purchases by *other* consumers depends on future prices.

Finally, notice that while we have only considered separating equilibria, it is fairly straightforward to construct pooling equilibria in which both types of sellers choose the same first-period price. A straightforward but tedious argument shows that increasing prices can occur at a pooling equilibrium as well.

## 5 Conclusion

We have constructed models of pricing a durable good or a new technology that confers network externalities. The models overturn the *price monotonicity property* that is a key element of Coasian dynamics: in each case discounted price rises over time.<sup>14</sup> In addition, discounted prices failed to rise whenever network externalities were not present, underscoring the close connection between introductory pricing and network externalities.

These results were derived in settings that were deliberately neutral toward introductory pricing. There is no cost escalation or growing demand that would justify increasing prices. Nor do our models allow for intertemporal competition that can result in low initial prices as in Katz and Shapiro (1985).

There is an interesting question not resolved in this paper: Given that network externalities can work to reverse the direction of Coasian dynamics, could they also refute the Coase conjecture itself? If our work is any indication, plausible demand and cost conditions may call for price to rise initially. Soon thereafter, however, the power of Coasian dynamics will likely prevail, causing prices to fall toward marginal cost. The possibility of such a price pattern remains an open issue.

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<sup>14</sup>The *skimming property* remained intact for those cases in which buyers are unambiguously rank ordered by willingness to pay.

## Appendix

**Proof of Proposition 1:** Suppose that sales occur in some period  $t$  at a price  $p_t$  in equilibrium. For contradiction, suppose that in an earlier period  $s < t$  the firm extends an offer of  $p_s < \delta^{t-s} p_t$ . Any buyer who purchased in period  $t$  would be better off by purchasing in period  $s$  instead. This would reduce her discounted outlay by  $\delta^t p_t - \delta^s p_s$  and also increase her utility by  $\sum_{\tau=s}^{t-1} \delta^\tau u_{i\tau}$  (both expressed in present value). As long as the buyer is small, this adjustment in purchasing behavior has no effect either on other buyers' utility or on the firm's profit. Therefore, every buyer who makes a purchase in period  $t$  will be induced to advance their purchases, so that in the end no sales will take place at time  $t$ . ■

**Proof of Proposition 2:** We begin by noting that Lemma 10.1 in Fudenberg and Tirole (1991) can be adapted to show that in any subgame in any period there exists a critical value  $v'$  such that buyers with valuations  $v \geq v'$  make a purchase in that period (if they haven't done so before). Specifically, denote by  $v_1$  the critical value in the first period and by  $v_2^k$  the two possible values in the second period depending on the number of first-period sales  $k = 0, 1$ . That is, consumers with valuation greater than  $v_1$  make a purchase in the first period; and consumers with valuation in  $[v_0^k, v_1]$  make a purchase in the second period. We establish, by construction, that the equilibrium is unique.

Begin by considering the second-period subgame that follows one sale in the first period. The indifferent buyer's valuation,  $v_2^1$  (period 2, 1 previous sale), is given by

$$u + v_2^1 = p_2^1, \quad (4)$$

where  $p_t^k$  is price in period  $t$  given that  $k$  sales have been made before. This gives the second-period profit function

$$\Pi_2^1 = p_2^1(v_1 - p_2^1 + u)/v_1, \quad (5)$$

where  $(v_1 - p_2^1 + u)/v_1$  is the equilibrium probability that a sale will occur in the second period given that a sale occurred in the first period. Substituting (4) for  $v_2^1$  in (5) and maximizing with respect to  $p_2^1$  yields

$$p_2^1 = (u + v_1)/2, \quad (6)$$

assuming that  $u < v_1$ . Substituting in (4), we get

$$v_2^1 = (v_1 - u)/2. \quad (7)$$

In order for this to be an interior solution, we require that  $v_2^1 > 0$ , which implies  $v_1 > u$ . Notice that, since  $v_2^k < v_1$ , the requirement that  $v_2^k < 1$  is implied by  $v_1 < 1$ , a condition which we will return to later.

Consider now the second-period subgame that follows no sales in the first period. The indifferent buyer's valuation,  $v_2^0$  (period 2, no previous sales), is now given by

$$v_2^0 + \frac{v_1 - v_2^0}{v_1}u = p_2^0. \quad (8)$$

Here  $(v_1 - v_2^0)/v_1$  is the probability that the other buyer will purchase conditional on not having done so earlier. Solving (8) for  $v_2^0$  yields

$$v_2^0 = v_1(p_2^0 - u)/(v_1 - u). \quad (9)$$

Therefore, the profit function is

$$\begin{aligned} \Pi_2^0 &= 2p_2^0(v_1 - v_2^0)/v_1 \\ &\quad + 2p_2^0 \left( v_1 - v_1(p_2^0 - u)/(v_1 - u) \right) / v_1 \\ &= 2p_2^0(v_1 - p_2^0)/(v_1 - u), \end{aligned} \quad (10)$$

where the second equality comes from substituting for  $v_2^0$  from (9). Maximization results in

$$p_2^0 = v_1/2, \quad (11)$$

which assumes  $u < v_1/2$ . Substituting (11) for price in (8) we get

$$v_2^0 = v_1 \frac{v_2/2 - u}{v_1 - u}. \quad (12)$$

To ensure that  $v_2^0 > 0$ , we must have  $v_1 > 2u$  which, in turn, requires that  $u < 1/2$ . For future reference, notice that

$$v_2^1 = v_2^0 + \frac{u^2/2}{v_1 - u}. \quad (13)$$

The second term in the right-hand side is positive, given the condition  $v_1 > u$ , and so  $v_2^1 > v_2^0$ . In words, the likelihood that a buyer will purchase in the second period is *higher* when no sales were made in the first period than if some had occurred.<sup>15</sup>

Having found the solution to both second-period subgames, we now turn to the first period. The indifferent buyer's valuation,  $v_1$ , is given by

$$v_1 + u \left( (1 - v_1) + \delta(v_1 - v_2^1) \right) - p_1 =$$

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<sup>15</sup>This results from two opposite effects. For a given second-period price, no sales in the first period implies lower probability of a sale in the second period (network externality effect). However, no sales in the first period also implies a lower price in the second period (price skimming effect). Both of these effects have, in general, the expected sign. Whichever is greater in absolute value depends on the particular shape of the distribution of  $v$ . We have just shown that, if the distribution is uniform, then the price skimming effect dominates the network externality effect in the second-period optimal price.

$$= \delta v_1 + \delta u(1 - v_2^0) - \delta \left( (1 - v_1)p_2^1 + \delta v_1 p_2^0 \right). \quad (14)$$

On the left-hand side, we have expected net utility from a purchase in the first period. For a price of  $p_1$ , the marginal buyer receives a standalone valuation  $v_1$  plus a network benefit when either the other buyer buys today (with probability  $1 - v_1$ ) or the other buyer buys tomorrow (with probability  $v_1 - v_2^1$ ). On the right-hand side we have expected net utility from a purchase in the second period. For an expected discounted price of  $\delta \left( (1 - v_1)p_2^1 + v_1 p_2^0 \right)$ , the marginal buyer receives a standalone valuation  $\delta v_1$  and an expected network externality of  $\delta u(1 - v_2^0)$ .

From (14) we can show that  $p_1$  is increasing in  $v_1$ , so that we can form the inverse demand function

$$p_1 = \delta(v_1 p_2^0 + (1 - v_1)p_2^1) + (1 - \delta)(v_1 + u(1 - v_1)) + \delta u(v_2^0 - v_2^1). \quad (15)$$

Expected discounted profit per buyer is given by

$$\Pi_1 = p_1(1 - v_1) + \delta(v_1(v_1 - v_2^0)p_2^0 + (1 - v_1)(v_1 - v_2^1)p_2^1), \quad (16)$$

where the factors multiplying the prices are the likelihoods of the three sales scenarios. Substituting for  $p_1$  from (15) and simplifying results in

$$\begin{aligned} \Pi_1 &= \delta(v_1(1 - v_2^0)p_2^0 + (1 - v_1)(1 - v_2^1)p_2^1) + \\ &\quad + (1 - v_1) \left( (1 - \delta)(v_1 + u(1 - v_1)) + \delta u(v_2^0 - v_2^1) \right) \\ &= \delta\Phi + (1 - v_1) \left( (1 - \delta)(v_1 + u(1 - v_1)) + \delta u(v_2^0 - v_2^1) \right), \end{aligned} \quad (17)$$

where we have isolated

$$\Phi \equiv v_1(1 - v_2^0)p_2^0 + (1 - v_1)(1 - v_2^1)p_2^1. \quad (18)$$

This last expression is just *ex ante* expected second-period price.

Next we differentiate (14) with respect to  $v_1$  and evaluate at  $v_1 = 1$  to get

$$\left. \frac{\partial \Pi_1}{\partial v_1} \right|_{v_1=1} = \delta \left. \frac{\partial \Phi}{\partial v_1} \right|_{v_1=1} - ((1 - \delta) + \delta u(v_2^0 - v_2^1)) \quad (19)$$

Setting  $\partial \Pi_1 / \partial v_1|_{v_1=1} = 0$  yields  $\delta = \bar{\delta}(u)$  as the solution. That is,  $\delta = \bar{\delta}(u)$  implies that the optimal  $v_1$  equals 1. Note that  $\bar{\delta}(u)$  is decreasing in  $u$ ; when  $u < 1/2$ ,  $\bar{\delta}(u)$  ranges from 1 down to  $16/17$ .

When  $v_1 = 1$ , no sales occur in the first period, and so discounted second-period price is simply  $\delta p_2^0$  with probability 1. When  $v_1 = 1$ , (15) reduces to

$$p_1 = \delta p_2^0 + (1 - \delta) + \delta u(v_2^0 - v_2^1). \quad (20)$$

In that case, the condition for introductory pricing to occur in equilibrium is just

$$(1 - \delta) + \delta u(v_2^0 - v_2^1) < 0, \quad (21)$$

where  $v_2^0$  and  $v_2^1$  are evaluated at  $v_1 = 1$ . Examining (19), this inequality reduces to

$$\left. \frac{\partial \Phi}{\partial v_1} \right|_{v_1=1} < 0. \quad (22)$$

Substituting (6), (7), (11), and (12) for  $p_2^1$ ,  $v_2^1$ ,  $p_2^0$ ,  $v_2^0$  in (18) and simplifying we get

$$\Phi = \frac{v_1^3(1 - u) - v_1^2(2 - u - u^2) - v_1(3 + u)u^2 + (2 + u)u^2}{4(u - v_1)} \quad (23)$$

Differentiating with respect to  $v_1$  and evaluating at  $v_1 = 1$  gives

$$\left. \frac{\partial \Phi}{\partial v_1} \right|_{v_1=1} = \frac{-u^4}{4(1 - u)^2} < 0, \quad (24)$$

ensuring that discounted price strictly increases with probability one.

Since the equilibrium value of  $v_1$  is an increasing function of  $\delta$ , it follows by continuity that if  $\delta$  is lower but sufficiently close to  $\bar{\delta}(u)$  then the equilibrium is interior (that is,  $v_1 < 1$ ) and discounted price strictly increases with probability one. ■

**Proof of Proposition 3:** First notice that each buyer's posterior regarding the value of  $\rho$ , depending on whether she is an  $H$  type or an  $L$  type, is given by

$$\rho^H = \frac{\rho \bar{\alpha}}{\rho \bar{\alpha} + (1 - \rho)\underline{\alpha}} \quad (25)$$

and

$$\rho^L = \frac{\rho(1 - \bar{\alpha})}{\rho(1 - \bar{\alpha}) + (1 - \rho)(1 - \underline{\alpha})}, \quad (26)$$

respectively. Comparing, we find that  $\rho^L < \rho < \rho^H$ . In words, being of the  $H$  type makes a buyer more optimistic about the measure of  $H$  types; conversely, being an  $L$  type makes the buyer more pessimistic about the measure of  $H$  types.

The seller has three possibilities: (1) induce both types to buy in the same period (pooling equilibrium), (2) induce  $H$  types to buy in the first period and  $L$  types in the second period, and lastly, (3) sell to  $H$  types only.

Consider first a pooling equilibrium. The highest price the seller can charge is given by the  $L$  types' expected utility, namely  $\rho^L \bar{\alpha} + (1 - \rho^L)\underline{\alpha}$ . However, since



$\underline{\alpha} < c$  by Condition (2), this price is lower than cost for a sufficiently low  $\rho$  in which case the firm would lose money.

Consider now a separating equilibrium. In the second period, the measure of adopters in the first period will be known. Based on this value, the seller and the remaining buyers will form a posterior on the value of  $\alpha$ . If  $\alpha = \bar{\alpha}$  (*i.e.*, the posterior puts weigh 1 on the value  $\alpha = \bar{\alpha}$ ), then the best the seller can do is to set  $p_2 = p_2(\bar{\alpha}) = \bar{\alpha}$ . It will then sell to the remaining  $1 - \bar{\alpha}$  consumers (recall that, by assumption,  $\bar{\alpha} > c$ ).

If, on the contrary,  $\alpha = \underline{\alpha}$  (*i.e.*, when  $\rho = 0$ ), then no price above cost will induce the  $L$  types to buy because, by assumption,  $\underline{\alpha} < c$ . In equilibrium, any price above  $\underline{\alpha}$  is indifferent from the seller's perspective. We take  $p_2 = p_2(\underline{\alpha}) = \underline{\alpha} + v$ , as this is the only price that survives the possibility that some  $H$  types might delay to the second period. Given our assumption that  $\underline{\alpha} + v > \bar{\alpha}$ , the second-period price is higher when  $\alpha = \underline{\alpha}$ , that is,  $p_2(\underline{\alpha}) > p_2(\bar{\alpha})$ .

Let us now consider the first-period price. The most the seller can charge and still have the high-valuation consumers make a purchase is given by

$$p_1 = \rho^H p_2(\bar{\alpha}) + (1 - \rho^H) p_2(\underline{\alpha}) \quad (27)$$

$$= \rho^H \bar{\alpha} + (1 - \rho^H)(\underline{\alpha} + v). \quad (28)$$

The justification for this is the following. The high-valuation consumers know that they will always want to make a purchase. Therefore, they only have to compare first-period price with expected second-period price *given their expectations about the value of  $\alpha$* .

Ex-ante expected second-period price, in turn, is given by  $\rho p_2(\bar{\alpha}) + (1 - \rho) p_2(\underline{\alpha})$ . But then,

$$\begin{aligned} E(p_2) &= \rho p_2(\bar{\alpha}) + (1 - \rho) p_2(\underline{\alpha}) \\ &> \rho^H p_2(\bar{\alpha}) + (1 - \rho^H) p_2(\underline{\alpha}) \\ &= p_1, \end{aligned} \quad (29)$$

where the inequality follows from the facts  $\rho^H > \rho$  and  $p_2(\underline{\alpha}) > p_2(\bar{\alpha})$ .

To conclude the proof, we have to check that the separating equilibrium is preferable to one in which the seller targets the  $H$ -type buyers only. In the latter equilibrium, the maximum price the seller can charge is given by the expected benefit for  $H$ -type buyers, that is

$$p_1 = \rho^H(\bar{\alpha} + v) + (1 - \rho^H)(\underline{\alpha} + v). \quad (30)$$

By comparison with (28), we can see that the separating equilibrium results in a loss of revenues from sales to  $H$ -type buyers given by  $\rho^H v$  (the difference in prices) times  $\rho \bar{\alpha} + (1 - \rho) \underline{\alpha}$  (the expected measure of  $H$  type buyers). Given the definition of  $\rho^H$ , this product is given by  $\rho \bar{\alpha} v$ . On the other hand, the separating equilibrium implies an extra profit from sales to  $L$ -type buyers, given by  $\rho(1 - \bar{\alpha})(\bar{\alpha} - c)$ . It is a simple exercise to check that Condition (3) implies that the separating equilibrium is preferable. ■

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