Strategic Commitments and the Principle of Reciprocity in Interconnection Pricing[1]

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Abstract

We discuss the effects of strategic commitments and of network size in the process of setting interconnection fees across competing networks. We also discuss the importance of the principles of reciprocity and imputation of interconnection charges on market equilibria. Reciprocity means that both networks charge the same for interconnection. Imputation means that a network charges its customers as much as it charges customers of the other network for the same service. Assuming that each consumer cannot subscribe to more than one network, we begin by analyzing a game of strategic symmetry where the two networks choose all prices simultaneously. Second, we allow a dominant network to set the interconnection fee before the opponent network can set its prices. This results in a price-squeeze on the rival network. Third, we show that the imposition of a reciprocity rule eliminates the strategic power of the first mover. Under reciprocity, one network sets the common interconnection fee at cost, and the equilibrium prices for final services are lower than in the two previous games without reciprocity. Moreover, prices under reciprocity obey the principle of imputation. In the long run, consumers subscribe to one of the two networks. Typically, there is a multiplicity of equilibria, including corner equilibria, where all consumers subscribe to the same network. However, under reciprocity, there are no corner equilibria.

JEL classification: L1, D4

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1 Introduction

It is a well-accepted fact of life that telephone networks are interconnected, so that a caller can reach anyone around the world. A typical phone call will pass through a number of networks owned by different firms. Each of these networks is paid an interconnection fee for allowing a call to pass though it or terminate in it. Who should pay whom, and how much, are difficult questions because of the complex nature of the interaction among networks.

Two networks may provide perfectly complementary services (i.e., combined in fixed proportions) or substitute services (i.e., only horizontally-related). Typically, however, networks are vertically related for some services and horizontally related for others.[1] For example, two networks that compete for provision of local access (and are thereby horizontally related) may also require each other's services to complete calls across the networks. A second important case arises when a bottleneck monopolist (say, of the local loop) also offers a service (say long distance or mobile telephony) on which it faces competition. In the presence of some horizontal relationship, the analysis of endogenous choice of compatibility[2] implies that interconnection should not be taken for granted, and in some cases firms may try to foreclose horizontally-related networks.[3] In other cases, a firm may interconnect but at high interconnection fees that result in a "price squeeze" of the rival network.[4]

In the present regulatory environment in the United States, interconnection fees are controlled by the Federal Communications Commission and State Public Utility Commissions. In a deregulated environment, such as New Zealand's, interconnection fees are hotly contested. It is this deregulated environment, which is emerging in the United States, that is the focus of our analysis. We model interactions among interconnected *two-way* networks, assuming that each consumer cannot subscribe to more than one network. Calls may originate in each network; thus, the issue is not primarily how to compensate the owner of a bottleneck facility for its use. Rather it is how to set termination fees for calls in interconnected networks, where it is anticipated that traffic will flow in both directions.[5]

[1]See Economides and White [10] and Economides and Salop [9].

[2]Economides [6] Matutes and Regibeau [15] Church and Gandal [4], Chou and Shy [3].

[3]See the historical evidence of AT&T foreclosures in Gabel and Weiman [13].

[4]Economides and Woroch [12].

[5]Thus the "efficient component pricing rule" (ECPR), developed under the assumption of a bottleneck facility, is not relevant for two-way networks. ECPR is proposed in Baumol and Sidak [1] and [2] and Willig [18]. For a critical view of the usefulness of the ECPR in bottleneck cases, see Economides and White [11].

We analyze the effects of monopoly power, including the possibility of a price squeeze by a dominant network through strategic commitments on interconnection fees. We also analyze the effects of two regulatory policies on interconnection pricing, reciprocity and imputation of interconnection charges on the market equilibrium. Reciprocity of interconnection fees requires that the termination fee set by network 1 is the same as the termination fee set by network 2.[6] Imputation of interconnection fees means that a network has to charge its customers the same amount it charges others for interconnection.

When the networks have equal size and set their prices simultaneously, we find that they charge equal interconnection fees to each other. Thus, reciprocity is a feature of the market equilibrium under symmetric conditions. However, a dominant (incumbent) network facing an entrant has a natural first mover's advantage in the termination fee, since the entrant has to accept an interconnection agreement to start business. This advantage translates into higher prices for incoming calls to the first mover.

The strategic advantage of the first mover is eliminated if firms are restricted to charge each other the same interconnection fee, i.e., if reciprocity is imposed. When reciprocity is not imposed, even under strategic symmetry, pricing exhibits "double marginalization," i.e., calls across networks are overpriced because each network fails to take into account the effects of its price changes on the opponent's profit. Reciprocity fully internalizes the vertical externality, thus eliminating the double marginalization and resulting in termination prices at cost, as well as in lower end-to-end prices. Thus, the application of the principle of reciprocity can improve social welfare.

In the long run, each consumer subscribes to (at most) one network. We show that, when reciprocity is not imposed, multiple subscription equilibria may exist, including corner equilibria where one of the networks has zero size. A first-mover advantage in setting termination fees typically results in a higher size for the first-moving network, although, corner equilibria may also exist. The imposition of reciprocity in termination pricing, however, eliminates the possibility of corner equilibria. Thus, the imposition of the *conduct* rule of reciprocity has significant *structural* effects. The structural effects of reciprocity are beneficial, since the eliminated corner equilibria have higher "transportation costs," and hence lower consumer surplus.

The rest of the paper is organized as follows. Section 2 sets up the network structure, derives the demand and profit functions, and discusses the various game structures that we consider. Section 3 characterizes all equilibria of the various game structures and compares

[6]The 1996 Telecommunications Act mandates that interconnection fees be based on reciprocal terms.

them. Section 4 presents welfare results. Section 5 contains extensions and generalizations. We conclude in section 6. Proofs are in the appendix.

2 The Model

2.1 Network Structure

Suppose that two firms ("networks"), $i = 1, 2$, offer local access for telephone services in the same area to a continuum of consumers. We assume that no consumer can subscribe to both networks. The two networks are interconnected, so that a customer of Network i can call any customer of Network j, as well as any customer of Network i.

Each phone call can be thought of as consisting of an originating part A_i and a terminating

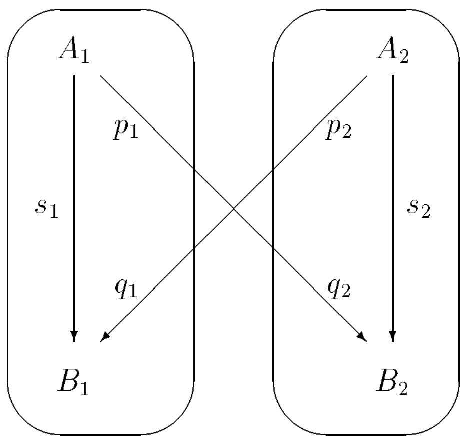

Figure 1: Network Structure

part B_j, where i and j refer to the identities of the networks of origination and termination. Each network i sets three prices: a price s_i for "internal" calls, i.e., calls that originate and terminate in the network; an origination fee p_i for "outgoing" calls., i.e., calls that originate in network i and terminate in the other network; and a termination fee q_i for "incoming" calls, i.e., calls that originate in the other network and terminate in network i. The four possible types of calls, with the corresponding prices as shown in Table 1.[7]

[7]It is not crucial whether the two component prices for calls across networks are paid directly by the consumer or the consumer pays the originating network for end-to-end service and the originating network buys termination services from the other network.

Call Type	Price charged by network 1	Price charged by network 2	Total price
Within network 1 "A_1B_1"	s_1	0	s_1
From network 1 to network 2, "A_1B_2"	p_1	q_2	$p_1 + q_2$
Within network 2 "A_2B_2"	0	s_2	s_2
From network 2 to network 1, "A_2B_1"	q_1	p_2	$p_2 + q_1$

Table 1

2.2 Demand and Profit Functions

Consumers perceive the two networks as horizontally (variety) differentiated; they are distributed uniformly according to their ideal network on the interval $[0, 1]$. The consumer who has the highest preference for network 1 (respectively 2) is "located" at point 0 (respectively 1).[8] Thus, a consumer of type $z \in [0, 1]$ who subscribes to network i derives total utility $V_z(i)$, where

$$V_z(1) = U_1 - \lambda z, \qquad V_z(2) = U_2 - \lambda(1 - z),$$

and U_i is her consumer surplus from buying telephone services from network i. The parameter $\lambda \in (0, \infty)$ measures the strength of preference for variety, i.e., the degree of perceived horizontal differentiation.

A consumer potentially makes calls to all other consumers. Denote by $x(\theta)$ the quantity of phone calls that she makes to consumer θ, where $\theta \in [0, 1]$. We assume that all consumers

[8] Differentiation in preferences of consumers across networks may arise when the networks have brand names that different consumers value differently, or if the networks use different technical specifications for which (business) customers equipment is more or less compatible. Tardiff (1995) reports evidence of brand loyalty toward long distance cariers.

have equal preferences over 'bundles' of telephone calls $\{x(\theta);\theta\in[0,1]\}$ and the outside good ("money") m. Preferences are represented by the quadratic functional

$$U(x,m)=\int_0^1\left(a\,x_\tau-\frac{b}{2}x_\tau^2\right)d\tau-\frac{c}{2}\int_0^1\int_0^1 x_\tau\,x_\theta\,d\tau\,d\theta+m,$$

for $0\leq x_\theta\leq\frac{a}{b}$, where a,b and c are positive real numbers, such that $c\in[0,b)$. The degree of substitutability between calls to any two different subscribers increases with c. When $c=0$, consumers have separable preferences, i.e., all phone calls are independent goods. In this case, our analysis can be extended to the case of general separable preferences:

$$U_s(x,m)=\int_0^1 u(x_\theta)\,d\theta+m,$$

where $u:[0,\infty)\to[0,\infty)$, with $u(0)=0$, $u'>0,u''<0$, and $2u''(y)+yu'''(y)\leq 0$.[9] Note that we have assumed that consumers derive no utility from receiving calls.

Let consumers in subset $N_i\subset[0,1]$ of measure n_i subscribe to network $i=1,2$. The budget constraint of subscriber θ of network 1 is

$$s_1\int_{N_1}x(\tau)\,d\tau+(p_1+q_2)\int_{N_2}x(\tau)\,d\tau+m=M_\theta,$$

where M_θ is her total wealth. Maximizing U subject to the budget constraint yields her demand function for "internal" calls x_{ii} (i.e., to any other customer of the same network) and her demand function for "outgoing" calls x_{ij} (i.e., to each customer of the other network:

$$x_{ii}(s_i,p_i+q_j;n_i,n_j)=\frac{1}{\gamma}\left(a-s_i+\frac{c}{b}n_j(p_i+q_j-s_i)\right),$$

$$x_{ij}(p_i+q_j,s_i;n_i,n_j)=\frac{1}{\gamma}\left(a-p_i-q_j+\frac{c}{b}n_i(s_i-p_i-q_j)\right),$$

where $\gamma\equiv b+c(n_1+n_2)$. Substituting these demands into the utility function yields the consumer surplus for each subscriber of network i:

$$U(s_i,p_i+q_j,n_i,n_j)=n_i\frac{(a-s_i)^2}{2\gamma}+n_j\frac{(a-p_i-q_j)^2}{2\gamma}+cn_in_j\frac{(p_i+q_j-s_i)^2}{2b\gamma}.$$

With separable preferences, maximizing U subject to the budget constraint yields the same demand function x, equal to the inverse of u', for both types of calls, independent of the network sizes n_i and n_j. Denoting $v(s)\equiv\max_x\{u(x)-sx\}$, we have

$$U_s(s_i,p_i+q_j,n_i,n_j)=n_iv(s_i)+n_jv(p_i+q_j).$$

[9] This last assumption on the third derivative guarantees that each network's marginal revenue is decreasing.

Thus, for both specifications of preferences, the model exhibits network externalities: given prices $s_i \in [0, a)$ and p_i and q_j such that $p_i + q_j \in [0, a)$, the welfare of each consumer is increasing in both n_i and n_j: i.e., the consumer derives positive externalities from expansion of each of the two networks. If the calls are independent goods, U and U_s are linear in n_i and n_j: each new subscriber increases the consumer's surplus by the same amount. Under non-separable preferences, U is strictly concave, since the addition of a new subscriber reduces the value of calling the other subscribers.

In the main part of the paper, we assume that the fixed costs are zero and normalize the marginal cost to zero. This assumption is relaxed in section 5.2.[10] Under non-separable preferences, firm i's profit function is[11]

$$\begin{aligned}\Pi_i(s_i, p_i, q_j, n_i, n_j) &= s_i\, n_i^2\, x_{ii}(s_i, p_i + q_j; n_i, n_j)\\ &\quad + p_i\, n_i\, n_j\, x_{ij}(p_i + q_j, s_i; n_i, n_j)\\ &\quad + q_i\, n_i\, n_j\, x_{ji}(p_j + q_i, s_j; n_i, n_j);\end{aligned}$$

and, with separable preferences, the firm's profit is

$$\Pi_i(s_i, p_i, q_j, n_i, n_j) = s_i\, n_i^2\, x(s_i) + p_i\, n_i\, n_j\, x(p_i + q_j) + q_i\, n_i\, n_j\, x(p_j + q_i).$$

In both cases, the three terms represent, respectively, the revenue from internal, outgoing, and incoming calls.

2.3 Game Structures

We model the interaction among the networks and the consumers as a two-stage game. In the first stage, all consumers simultaneously make their subscription decisions. In the second stage, the networks set their prices, and the consumers choose their consumption levels. Thus, the consumers cannot change their subscription decision after observing the networks' prices.[12]

This game structure aims at capturing situations where consumers are slower in changing network affiliation than in varying the amount of phone calls they make as firms change prices:

[10] We assume that investment costs are zero. Investment costs that depend on network size, $F_i(n_i)$, would play a similar role as the parameter λ.

[11] The aggregate demand functions are: $D_{ii} = n_i^2 x_{ii}$, and $D_{ij} = n_i n_j x_{ij}$,where $i, j = 1, 2, i \neq j$.

[12] This does not mean that, when making the subscription decision, the consumers are uncertain about the prices set by the networks in the second stage: in equilibrium, they anticipate correctly all other parties' actions.

one can think of the second stage of the game as the "short run," and of the first stage as the "long run".

We analyze three alternative structures for the second stage. First, in the benchmark structure, there is strategic symmetry: i.e., the firms set all six prices simultaneously. Second, we analyze a game where one firm (firm 1) sets its interconnection fee q_1 in advance. This structure captures situations where a dominant network is able to set its interconnection charge before the other network has a chance to play. This happens, for example, when there is a single incumbent, and an entrant needs an interconnection agreement (specifying the termination fee) before starting business. Finally, in the third game, firm 1 chooses the interconnection fee under the constraint of reciprocity in termination fees, i.e., $q_1 = q_2$.

We analyze both the case where the interconnection fee is set before the other prices, and the case where all six prices are set simultaneously. Reciprocity is imposed by law in many but not all jurisdictions. For example, in the United States reciprocal pricing of call termination is mandated by the Telecommunications Act of 1996, section 251(b)(5). On the other hand, the law is silent on reciprocity in New Zealand, and the issue of reciprocal termination pricing is central in the negotiations between telecommunications service providers.

3 Analysis

3.1 Game 1: Strategic Symmetry

3.1.1 Equilibrium Prices

To find the subgame perfect equilibria, we start by solving by backward induction. In the second stage, the networks set their prices simultaneously, given their sizes n_1 and n_2. The next proposition characterizes the equilibrium prices.

Proposition 1 *With general separable preferences, for $n_1 > 0,\ n_2 > 0$, the equilibrium prices are*

$$s_i^{(1)} = s^m, \quad p_i^{(1)} = q_i^{(1)} = \frac{t^o}{2}, \quad i = 1, 2,$$

where $x(s^m) + s^m x'(s^m) \equiv 0$ and $x(t^o) + \frac{t^o}{2} x'(t^o) \equiv 0$. Moreover, $s^m < t^o$. If $n_j = 0$, then $s_i^{(1)} = s^m$ and all other prices can take arbitrary values.

With non-separable quadratic preferences, for $n_1 > 0,\ n_2 > 0$, the equilibrium prices are:

$$s_i^{(1)} = \frac{a}{2}, \qquad p_i^{(1)}(n_i) = \frac{a(2b + 3cn_i)}{6(b + cn_i)}, \qquad q_j^{(1)}(n_i) = \frac{ab}{3(b + cn_i)}; \ \ i = 1, 2.$$

In $n_j = 0$, then $s_i^{(1)} = \frac{a}{2}$ and all other prices can take arbitrary values.

PROOF. See appendix. □

A number of observations are in order. First, under both preferences specifications, and for any network size $n_i \in (0,1)$, $s_i^{(1)} < p_i^{(1)} + q_j^{(1)}$; that is, outgoing calls are sold at a higher price than internal calls.[13] This result is due to the fact that, while each network i supplies both components of its internal calls (the originating part A_i and the terminating part B_i), the two components of any outgoing call are sold by different networks. In the price-setting process for outgoing calls, each firms fails to internalize the full benefit of a reduction in the price of its components. Thus, the perceived elasticity of demand is lower, hence the equilibrium total price $p_i + q_j$ is higher than the joint monopoly profit-maximizing price.[14]

Second, in general, imputation fails to occur; i.e., the larger network charges more for its origination and termination services when they are sold as part of hybrid calls than when they are used by itself, i.e., if $n_i > n_j$, then $s_i < p_i + q_i$.

Moreover, under non-separable preferences, we have the following additional results:

Third, the origination fee of an outgoing phone call is always larger than the termination fee of the same call, $p_i^{(1)} > q_j^{(1)}$. This is because the originating network has a strategic incentive to keep the price of outgoing calls high, since they are substitutes with its internal calls. On the other hand, the terminating network has no *strategic* incentive to keep termination prices high, since the incoming call is not a substitute for its internal calls or for outgoing calls that originate from it.

Fourth, the equilibrium origination fee $p_i^{(1)}$ and termination fee $q_j^{(1)}$ for outgoing calls are respectively increasing and decreasing functions of the originating (i) network's size, $dp_i^{(1)}/dn_i > 0$, $dq_j^{(1)}/dn_i < 0$, while the price of outgoing calls decreases in the size of the originating network, $d(p_i^{(1)} + q_j^{(1)})/dn_i < 0$. These are all consequences of the relative strategic strengths of the two networks. As the size of network i increases, its stronger strategic power is reflected in a higher origination fee; this prompts a sharply lower termination fee by the opponent network, so that a hybrid call has a lower price despite the increase in its origination fee.

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[13] To see that $s^m < t^o$, let $m(z) = x(z) + zx'(z)$ and $M(z) = x(z) + zx'(z)/2$. The result follows immediately from the fact that $M(z)$ and $m(z)$ are monotone and $M(z) - m(z) = -zx'(z)/2 > 0$.

[14] This effect was noted by Cournot [5] in a simpler model with only two complementary components. For an application to network industries see Economides [6]. The problem is similar to the "double marginalization" problem that arises when a single good is produced by a manufacturer and sold by a retailer (Spengler [17].)

Fifth, as a consequence of the inequalities stated above, if network i is larger than network j, $n_i > n_j$, its outgoing calls are offered at a lower price, $p_i + q_j < p_j + q_i$, but its origination and termination fees are higher, $p_i > p_j$, $q_i > q_j$. Therefore, if network sizes differ, $n_i \neq n_j$, reciprocity fails.

3.1.2 The Subscription Decision

We now turn to the analysis of the consumers' subscription decisions. In equilibrium, each consumer makes her choice, correctly anticipating the simultaneous choices of all other consumers as well as the prices that the firms will set in the second stage, as given in Proposition 1. When the consumers in subset N_i of measure n_i subscribe to network i, $i = 1, 2$, the overall realized utility of the consumer located at point θ, is $V^{(1)}(n_1, n_2) - \lambda\theta$, if she subscribes to network 1 and $V^{(1)}(n_2, n_1) - \lambda(1-\theta)$ if she subscribes to network 2, where[15]

$$V^{(1)}(n_i, n_j) \equiv U\left(s_i^{(1)}, p_i^{(1)}(n_i) + q_j^{(1)}(n_i), n_i, n_j\right),$$

and similarly for the separable case. Thus, in the non-separable case,

$$V^{(1)}(n_i, n_j) = \frac{a^2\left(4bn_j + 9bn_i + 9cn_in_j + 9cn_i^2\right)}{72\gamma\left(b + cn_i\right)},$$

and, in the separable case,

$$V_s^{(1)}(n_i, n_j) = n_i v(s^m) + n_j v(t^o).$$

For both preference specifications, the consumer welfare increases with both network sizes: i.e., $\frac{\partial}{\partial n_i} V^{(1)}(n_i, n_j)$, $\frac{\partial}{\partial n_i} V_s^{(1)}(n_i, n_j)$, $\frac{\partial}{\partial n_j} V^{(1)}(n_i, n_j)$ and $\frac{\partial}{\partial n_j} V_s^{(1)}(n_i, n_j)$ are all positive. Thus, the market-mediated indirect utility function exhibits network externalities.

The next proposition characterizes all equilibria of game 1. An equilibrium is indicated as a pair $\{n_1, n_2\}$ of network sizes.

Proposition 2 *For all parameter values and both consumer preferences specifications,* $\{0, 0\}$ *is an equilibrium. In addition, with non-separable quadratic preferences, the equilibrium correspondence is determined by five numbers* $\lambda_1^{(1)} < \ldots < \lambda_5^{(1)}$ *(defined in the appendix) as follows:*

- $\left\{\left\{\frac{1}{2}, \frac{1}{2}\right\}, \{0, 1\}, \{1, 0\}\right\}$, *for* $0 < \lambda \leq \lambda_1^{(1)}$;

[15] $V^{(1)}$ is well defined for $n_i \in (0, 1]$. For $n_i = 0$, $V^{(1)}$ depends on $p_i + q_j$, which are non-uniquely determined.

- $\left\{\left\{\frac{1}{2},\frac{1}{2}\right\},\{0,1\},\{1,0\},\left\{\frac{1}{2}-\delta(\lambda),\frac{1}{2}+\delta(\lambda)\right\},\left\{\frac{1}{2}+\delta(\lambda),\frac{1}{2}-\delta(\lambda)\right\}\right\}$, *where* $\delta(\lambda) \equiv \sqrt{\frac{b}{c^2}\left(\frac{(c+2b)^2}{4b}-\frac{5a^2}{72\lambda}\right)}$, *for* $\lambda_1^{(1)} < \lambda < \lambda_2^{(1)}$;
- $\left\{\left\{\frac{1}{2},\frac{1}{2}\right\},\{0,1\},\{1,0\}\right\}$, *for* $\lambda_2^{(1)} \leq \lambda \leq \lambda_3^{(1)}$;
- $\left\{\left\{\frac{1}{2},\frac{1}{2}\right\}\right\}$, *for* $\lambda_3^{(1)} \leq \lambda \leq \lambda_4^{(1)}$; *and*
- $\left\{\left\{n_*^{(1)}(\lambda), n_*^{(1)}(\lambda)\right\}\right\}$, *where*

$$n_*^{(1)}(\lambda) \equiv \frac{1}{48c\lambda}\left(3a^2-36\lambda b+\sqrt{(9a^4-8a^2\lambda b+144\lambda^2 b^2)}\right),$$

  for $\lambda_4^{(1)} < \lambda < \lambda_5^{(1)}$.

With separable preferences, the equilibrium correspondence consists of

- $\left\{\left\{\frac{1}{2},\frac{1}{2}\right\},\{0,1\},\{1,0\}\right\}$, *for* $0 < \lambda < v(s^m) - v(t^o)$;
- $\{\{n, 1-n\}; 0 \leq n \leq 1\}$, *for* $\lambda = v(s^m) - v(t^o)$,
- $\left\{\left\{\frac{1}{2},\frac{1}{2}\right\},\{0,1\},\{1,0\}\right\}$, *for* $v(s^m) - v(t^o) < \lambda < v(s^m)$;
- $\left\{\left\{\frac{1}{2},\frac{1}{2}\right\}\right\}$, *for* $v(s^m) < \lambda < v(s^m) + v(t^o)$; *and*
- $\left\{\{n,n\}; 0 \leq n \leq \frac{1}{2}\right\}$, *for* $\lambda = v(s^m) + v(t^o)$.

Proof. See appendix. □

Proposition 2 establishes that, except when λ is large ($\lambda > \lambda_5^{(1)}$), game 1 has multiple equilibria. However, imposing the requirement that the equilibria satisfy a notion of stability (see below), restricts the equilibrium set as follows: in the separable case, only one equilibrium is stable for almost all parameter values; under non-separable preferences, there is a unique stable equilibrium except for $\lambda \in \left[\lambda_1^{(1)}, \lambda_2^{(1)}\right]$, where both the corner equilibria and the symmetric equilibrium are stable. To define the stability notion, suppose that each consumer assigns a positive probability to the event that a (small, but of positive measure) fraction of consumers do not make their equilibrium subscription and that, if the corner outcome $n_i = 1 - n_j = 1$ occurs, the firm will set prices $p_i(1) = \frac{a(2b+c)}{6(b+c)}$, $q_j(1) = \frac{ab}{3(b+c)}$, and $p_j(0) = q_i(0) = \frac{a}{3}$. We say that an equilibrium is "unstable" if, in this case, some consumers have an incentive to revise their choice.

According to this notion, $\{0,0\}$ is unstable whenever another equilibrium exists. Also, under separable preferences, $\left\{\frac{1}{2},\frac{1}{2}\right\}$ is unstable for $\lambda < v(s^m) - v(t^o)$; and the corner equilibria are unstable for $v(s^m) - v(t^o) < \lambda < v(s^m)$. Thus, neglecting the knife-edge cases where $\lambda \in \{v(s^m) - v(t^o), v(s^m) + v(t^o)\}$, the only 'stable' equilibria are the corner ones for $\lambda < v(s^m) - v(t^o)$ and the symmetric one for $v(s^m) - v(t^o) < \lambda$.

Under non-separable preferences, the stability notion eliminates the symmetric equilibrium for $\lambda < \lambda_1^{(1)}$, the two interior asymmetric equilibria whenever they exist (i.e., for $\lambda_1^{(1)} < \lambda < \lambda_2^{(1)}$,) and the corner equilibria for $\lambda_2^{(1)} < \lambda < \lambda_3^{(1)}$.

To interpret the structure of the equilibrium correspondence, note the forces that determine them. First, consumers want to belong to a large network. Second, the benefit to a consumer of joining a network is diminished by the loss of utility which this consumer incurs because the prospective network does not coincide with her "most-preferred" network specification. This "horizontal differentiation" cost is measured by λ. Each network's size is determined by its marginal consumer, who, in equilibrium, must prefer joining her chosen network to both joining the other network and not joining any network. Thus, different values of λ imply different equilibria. If the preferences for variety are not strong, i.e., $\lambda \in \left(0, \lambda_1^{(1)}\right)$, the incentive to congregate to a single network dominates. Hence the corner equilibria exist and the symmetric equilibrium is unstable. As λ increases and enters the interval $\left(\lambda_1^{(1)}, \lambda_2^{(1)}\right)$, the symmetric equilibrium becomes stable. In the interval $\left(\lambda_2^{(1)}, \lambda_4^{(1)}\right)$, the are no stable corner equilibria, and the unique stable equilibrium is the symmetric one. Finally, for $\lambda \in \left(\lambda_4^{(1)}, \lambda_5^{(1)}\right)$, we have a unique, symmetric equilibrium with partial coverage. Full coverage equilibria disappear since, for the consumer located at $\frac{1}{2}$, the horizontal differentiation cost now outweighs the net benefit from joining any network. As λ increases further, the size of each network shrinks to zero. Eventually, for $\lambda \geq \lambda_5^{(1)}$, $\{0,0\}$ remains the only equilibrium.

3.2 Game 2: Commitment by One Network on the Termination Fee

3.2.1 Equilibrium Prices

Given the network sizes from stage 1, in this game, pricing takes place sequentially. First, firm 1 sets its termination fee q_1. Then both firms set all other prices simultaneously. For simplicity, we restrict the analysis to the case of quadratic preferences, including the (separable) case where $c = 0$. In the short-run, firm one chooses the interconnection fee before its opponent.

Proposition 3 *In game 2, with quadratic preferences, the equilibrium prices* $s_1^{(2)}$, $s_2^{(2)}$, $p_1^{(2)}$ *and* $q_2^{(2)}$ *are equal to the corresponding ones in game 1. Moreover,*

$$p_2^{(2)} = a\frac{b+2cn_2}{4\left(b+cn_2\right)} < p_2^{(1)}, \quad \textit{and} \quad q_1^{(2)} = \frac{ab}{2\left(b+cn_2\right)} > q_1^{(1)}.$$

Further, $p_2^{(2)} + q_1^{(2)} > p_2^{(1)} + q_1^{(1)}$, *and* $p_1^{(2)} > p_2^{(2)}$, $q_1^{(2)} > q_2^{(2)}$.

Proof. Straightforward. □

A number of observations are in order. First, the strategic advantage of being able to commit on the interconnection fee allows firm 1 to charge higher origination and termination fees than the opponent, $p_1^{(2)} > p_2^{(2)}$, $q_1^{(2)} > q_2^{(2)}$, for any network sizes in the separable case as well as when the two networks are of equal sizes in the non-separable case; thus reciprocity fails. Under the same conditions, outgoing calls from network 1 are cheaper than outgoing calls from network 2, $p_2^{(2)} + q_1^{(2)} > p_1^{(2)} + q_2^{(2)}$. These result from the strategic advantage of the leader.

Imputation also fails in general. The leader always prices its origination and termination components higher to others than to itself, i.e., $s_1^{(2)} < p_1^{(2)} + q_1^{(2)}$. On the other hand, the follower may price its components lower to others than to itself.

A number of the qualitative results of the simultaneous game are preserved. Internal calls are cheaper than outgoing calls. Origination fees are increasing in the size of the originating network, while termination fees and total fees for outgoing calls are decreasing in the size of the originating network.

Finally, the customers of network 1 face the same prices as in game 1: hence their welfare remains unchanged. The customers of network 2 face a higher price for their outgoing calls, (and the same price for the internal calls): thus their surplus is lower than in game 1. It follows that total consumer surplus is lower in game 2.

3.2.2 The Subscription Decision

In the first stage, the consumers make their subscription decisions. The next proposition characterizes all equilibria $\{n_1, n_2\}$ for the separable utility case.

Proposition 4 *The equilibrium correspondence is as follows:*

- $\left\{\{1,0\}, \left\{n_*^{(2)}(\lambda), 1-n_*^{(2)}(\lambda)\right\}, \{0,1\}\right\}$, *where* $n_*^{(2)}(\lambda) \equiv 4\frac{5a^2-72\lambda b}{47a^2-576\lambda b}$, *for* $0 < \lambda \leq \frac{5a^2}{72b}$;

- $\{\{1,0\}\}$, *for* $\frac{5a^2}{72b} < \lambda < \frac{3a^2}{32b}$;
- $\left\{\left\{n_*^{(2)}(\lambda), 1 - n_*^{(2)}(\lambda)\right\}\right\}$ *for* $\frac{3a^2}{32b} < \lambda < \frac{a^2}{6b}$;

PROOF. See Appendix. □

Notice that symmetric equilibria, $n_1 = n_2 = 1/2$, never exist. Unique asymmetric interior equilibria with full coverage exist in two separate regions of λ. When λ is large, the full coverage equilibrium is stable and network 1 is larger, benefiting from its first mover advantage. When λ is small, the full coverage equilibrium is unstable and network 2 is larger. It is likely that such equilibria will not be observed.

3.3 Game 3: Commitment in the Termination Fee with Reciprocity

We now consider the case when network 1 chooses the interconnection fee subject to reciprocity: $q_1 = q_2 = q$. Thus, firm 1 is unable to create a difference in interconnection fees to its advantage, although it has control over its rival's termination fee. We analyze two game structures. In the first (game 3.1), network 1 sets $q_1 = q_2 = q$, s_1, and $p_1$and, simultaneously, firm 2 chooses s_2 and p_2. The second game structure (game 3.2) has one additional stage. Network 1 chooses q in advance; subsequently network 1 chooses s_1 and p_1, and network 2 chooses s_2 and p_2.

Proposition 5 *In both games 3.1 and 3.2, the equilibrium prices are as follows: in the non-separable case,*

$$q = 0, \quad s_i = p_i = \frac{a}{2}, \quad i = 1, 2; \tag{1}$$

and, in the separable case,

$$q = 0, \quad s_i = p_i = t^m, \quad i = 1, 2, \tag{2}$$

where $x(t^m) + t^m x'(t^m) \equiv 0$.

PROOF. Straightforward. □

Proposition 5 shows that, under reciprocity on the termination fees, network 1 sets the interconnection fee equal to its marginal cost (zero) and both networks set the other two prices at the monopoly level. This happens independently of whether firm 1 sets the interconnection fee in advance or simultaneously with all the other prices. Thus, imposing reciprocity eliminates the "double marginalization" effect: the firms charge their monopoly prices on both internal and outgoing calls. In comparison to game 1, the welfare of all consumers, as well as both firms' profits are higher. Note also that reciprocity implies exact imputation, that is, at equilibrium, $s_i = p_i + q_i$.

The intuition behind the results is as follows. The reciprocity constraint enables network 1 to control the *total* price $p_1 + q$ of its outgoing calls. Thus, network 1 is able to fully reap the benefits of price decreases of components of $A_1 B_2$, thereby eliminating the "double marginalization" effect. Network 1's sales of components A_1 and B_1 become equivalent to sales of components A_1 and B_2. Thus, network 1 acts as a monopolist on both its internal and its outgoing calls. Looking at the separable case for simplicity, network 1's problem for outgoing calls is:

$$\max_q \ (p^o(q) + q)\ x(p^o(q) + q)$$

where $p^o(q) = \arg\max_p \ p\ x(p+q)$. The optimal price for outgoing calls is then the monopoly price t^m, which can be written as $t^m = p^o(q^*) + q^*$. This monopoly price can only be achieved (in both the simultaneous and sequential structures) by setting $q = q^* = 0$, since $p^o(0) = t^m$ and $p^o(q)$ is strictly increasing. In other words, first, reciprocity allows network 1 to achieve the monopoly pricing for outgoing calls; second, monopoly pricing can only occur when the first markup is zero. Thus, network 1 sets the termination fee at zero.

3.3.1 The Subscription Decision

Turning to the consumers' subscription decision, each consumer earns the same surplus independent of her network affiliation, since internal calls cost as much as outgoing calls. In the non-separable case,

$$V^{(3)}(n_i, n_j) = \frac{a^2(n_i + n_j)}{8(b + c(n_i + n_j))};$$

and in the separable case:

$$V^{(3)}(n_i, n_j) = (n_i + n_j)\, v(t^m).$$

Proposition 6 *In the non-separable quadratic case, in addition to* $\{0,0\}$, *the equilibria are*

- $\left\{\left\{\frac{1}{2}, \frac{1}{2}\right\}\right\}$ *for* $0 < \lambda \le \frac{a^2}{4(b+c)}$;
- $\left\{n_*^{(3)}(\lambda), n_*^{(3)}(\lambda)\right\}$, *for* $\frac{a^2}{4(b+c)} < \lambda \le \frac{a^2}{4b}$, *where* $n_*^{(3)}(\lambda) \equiv \frac{a^2 - 4\lambda b}{8\lambda c}$.

In the separable case, in addition to $\{0,0\}$, *there is only one other equilibrium,* $\left\{\frac{1}{2}, \frac{1}{2}\right\}$, *for* $0 < \lambda \le 2v(s^m)$.

PROOF. See appendix. □

Thus, *reciprocity eliminates the corner equilibria* which may arise in both games 1 and 2; the symmetric one is the only full coverage equilibrium. In the price subgame, reciprocity eliminates the power of the leader to set different prices for termination, and the leader finds it to its benefit to set zero termination charges, resulting in equal prices for outgoing and internal calls. This eliminates corner equilibria which require high prices for outgoing calls. Thus, reciprocity — a conduct rule — has a structural effect, the elimination of corner equilibria and the promotion of duopoly over monopoly.

4 Welfare analysis

The following table summarizes the welfare analysis of games 1 and 3 for the quadratic case. Let CS denote the total consumer surplus, and Π denote the sum of the profits of the two networks.

	Game 1 (symm. eq.)		Game 1 (corner eq.)		Game 3 (reciprocity)
CS	$\frac{a^2(13b+9c)}{72(b+c)(2b+c)} - \frac{\lambda}{4}$	$<$	$\frac{a^2}{8(b+c)} - \frac{\lambda}{2}$	$<$	$\frac{a^2}{8(b+c)} - \frac{\lambda}{4}$
Π	$\frac{a^2(17b+9c)}{36(b+c)(2b+c)}$	$<$	$\frac{a^2}{4(b+c)}$	$=$	$\frac{a^2}{4(b+c)}$

The rankings in the table hold for any value of λ such that the equilibria exist. The profits' ranking is a consequence of the "double marginalization" effect, which is present only in the symmetric equilibrium of game 1. At the corner equilibria, only one firm is producing; hence there is no demand for outgoing calls; and in game 3, the reciprocity constraint eliminates the double marginalization problem by incorporating the termination fee in firm 1's decision problem.

The elimination of double marginalization also increases total consumer surplus, since it lowers the equilibrium prices to their monopoly level. This happens both at the corner equilibria of game 1 and at the equilibrium of game 3. The latter, however, is preferable from the consumers' perspective, since their total "transportation costs" are minimized at the symmetric outcome.

5 Extensions

5.1 Heterogeneous preferences

In this section, we show that the results obtained for the short run in the previous sections hold even if consumers have heterogeneous preferences for telephone services, provided that these preferences are not correlated with their preferences over network variety. In other words, the critical condition is that each consumer's position on the unit segment is independent of her preferences for telephone services.

In the model of section 2, each consumer has the same preferences over telephone consumption. One way in which this assumption can be generalized is to assume that the consumer located at $\theta \in [0,1]$ has utility function

$$U^{\psi}(x,m) = \psi(\theta)\left[\int_0^1 \left(a\,x_\tau - \frac{b}{2}x_\tau^2\right)d\tau - \frac{c}{2}\int_0^1\int_0^1 x_\tau\,x_{\tau'}\,d\tau\,d\tau'\right] + m,$$

where ψ is any integrable function defined on $[0,1]$. $\psi(\theta)$ measures the intensity of preference for telecommunications services for a consumer of type θ. The corresponding demand functions are

$$\begin{aligned} x_{ii}^{\psi} &= \frac{\psi(\theta)}{\gamma}\left(a - s_i + \frac{c}{b}n_j\left(p_i + q_j - s_i\right)\right), \\ x_{ij}^{\psi} &= \frac{\psi(\theta)}{\gamma}\left(a - p_i - q_j + \frac{c}{b}n_i\left(s_i - p_i - q_j\right)\right), \end{aligned}$$

and the profit functions are

$$\begin{aligned} \Pi_i^{\psi} &= m_i\,n_i\,s_i\frac{1}{\gamma}\left(a - s_i + \frac{c}{b}n_j\left(p_i + q_j - s_i\right)\right) \\ &\quad + m_i\,n_j\,p_i\frac{1}{\gamma}\left(a - p_i - q_j + \frac{c}{b}n_i\left(s_i - p_i - q_j\right)\right) \\ &\quad + m_j\,n_i\,q_i\frac{1}{\gamma}\left(a - p_j - q_i + \frac{c}{b}n_j\left(s_j - p_j - q_i\right)\right), \end{aligned}$$

where $m_i \equiv \int_{N_i}\psi(\theta)\,d\theta \equiv n_i\,E\left[\psi(\theta)\,|\theta \in N_i\right] \equiv n_i\,\psi_i$.

With these preferences, the short-run equilibrium outcome in game 1 remains unchanged. In game 3, we still have $s_i^{(3)} = \frac{a}{2}$, and $p_i^{(3)} = \frac{1}{2}(a-q)$, as in the case of identical preferences. Network 1 now chooses

$$q^{(3)} = \frac{ab\left(\psi_2 - \psi_1\right)}{2\psi_2\left(b + cn_2\right) - \psi_1\left(b + cn_1\right)}.$$

Thus $q^{(3)} = 0$ if and only if $\psi_2 = \psi_1$, which holds if consumers' intensity of preference for telephone calls (represented by the function ψ) are not correlated with their preferences for variety (represented by their position θ on the unit segment.)

5.2 Different costs

In the main part of the paper we assumed that the costs of the two networks were the same, and without further loss of generality we took them to be zero. This assumption is reasonable if the two networks operate in the same area and face the same geographic conditions, given that the technology of production is typically well known. However, reciprocity has also been proposed and practiced in international telephony, where the costs can easily differ across the two networks (countries). Of course, if the two networks are at different locations, the subscription problem is not relevant. This section investigates the effects of reciprocity when marginal production costs differ across networks. We show that the regulatory imposition of a generalized reciprocity rule has the same effects as in the equal costs case. The generalized reciprocity rule takes the form of equal markups above marginal costs, and we call it "reciprocity in markups".

For simplicity, we only show here the case with linear demands and independent goods; the proof for general demand is identical. Assume that network i's marginal cost of providing either origination or termination services is m_i, $i = 1, 2$. Then firm i's profit Π_i satisfies

$$\begin{aligned} b\Pi_i &= (s_i - 2m_i)\, n_i^2\, (a - s_i) \\ &\quad + (p_i - m_i)\, n_i n_j\, (a - p_i - q_j) \\ &\quad + (q_i - m_i)\, n_i n_j\, (a - p_j - q_i)\,. \end{aligned}$$

First, as a benchmark, note that the prices that maximize the joint profits $\Pi_1 + \Pi_2$ are:

$$s_i = \frac{a}{2} + m_i, \quad \text{and} \quad p_i + q_j = \frac{1}{2}(a + m_1 + m_2)\,.$$

Solving for the Nash equilibrium of the strategic symmetry price subgame (Game 1), yields:

$$s_i = \frac{a}{2} + m_i, \quad p_i = q_i = \frac{1}{3}(a - m_j + 2m_i)$$

and

$$p_i + q_j = \frac{1}{3}(2a + m_1 + m_2)\,.$$

Thus, in Game 1, at the Nash equilibrium, $p_i + q_j$ is higher than the joint profit maximizing level. As before, this is due to the "double marginalization" effect, i.e., to the failure of each network to internalize the full effect of changing its prices.

The generalized reciprocity rule is applied to markups; i.e., it is imposed that the markup above cost of network 1 be equal to the markup above cost of network 2:

$$q_2 - m_2 = q_1 - m_1.$$

Maximizing Π_1, subject to this constraint, with respect to s_1, p_1 and q_1, maximizing Π_2 with respect to s_2 and p_2, and solving for the equilibrium yields

$$s_i = \frac{a}{2} + m_i, \quad p_j = \frac{1}{2}(a - m_i + m_j), \quad \text{and} \quad q_i = m_i,$$

for $i = 1, 2$. Therefore the imposition of reciprocity on markups results in pricing of termination at cost. It follows that outgoing calls are priced at

$$p_i + q_j = \frac{1}{2}(a + m_1 + m_2),$$

and therefore all prices are as in the collusive outcome. This is because, imposing "reciprocity in markups" on the termination fees, eliminates the double marginalization distortions. Note, however, that imputation fails, $p_i + q_i \neq s_i$, unless marginal costs are equal across networks.

5.3 Low switching costs

Up to this point, we have assumed that the two networks set their prices only after the consumers make irrevocable subscription decisions. This feature of our model aims to capture the idea that changing network affiliation is costly in a particular way: it is only feasible in the "long-run." We believe that our basic model is close to reality, as telecommunications providers have observed that consumers are slow to change network affiliation.[16] However, to test our results to alternative specifications, in this subsection, we allow for simultaneous subscription and quantity decisions by the consumers, which follow the announcement of prices.

The analysis with low switching costs is complicated by the presence of network externalities. After firms have chosen prices, consumers will choose different quantities of output as well as network affiliation depending on what each consumer believes the other consumers

[16] See Radner [16].

will do. Thus, the demand function faced by a network, as well as its size, depends on coordination among the consumers in the subgame. This makes each firm's maximization problem dependent on its conjectures about the consumers' choices in the subgame starting after the firms set their prices. Moreover, in setting prices, it is natural to expect that a network will take actions to tilt the coordination of the consumers in its favor. Thus, the problem with low switching costs is considerably more complex than the one of high switching costs. We are able, however, to establish the existence of corner equilibria when consumers have a weak preference for variety.

Proposition 7 *Consider the following multi-stage games:*

Game 1': in stage 1*, the networks set their three respective prices simultaneously; in stage* 2 *the consumers make their subscription and consumption decisions;*

Game 2': in stage 1*, network* 1 *chooses* q_1*; in stage* 2*, the two networks set all other prices simultaneously; in stage* 3*, the consumers make their subscription and consumption decisions.*

Game 3.1': in stage 1*, network* 1 *chooses* $q_1 = q_2$*; in stage* 2*, network* 2 *sets* s_2*, and* p_2*, and, simultaneously, network 1 sets* s_1 *and* p_1*; in stage* 3*, the consumers make their subscription and consumption decisions.*

Game 3.2': in stage 1*, network* 2 *sets* s_2*, and* p_2*, and, simultaneously, network 1 sets* s_1*,* p_1*,* q_1 *and* q_2*, subject to* $q_1 = q_2$*; in stage* 2*, the consumers make their subscription and consumption decisions.*

In all four games above, with quadratic preferences, corner equilibria exist, where $n_i = 1 - n_j = 0$*,* $s_i = \frac{a}{2}$*,* $p_i = 0$ *and* $q_i > a$*, for all* $\lambda \leq \frac{a^2}{8(b+c)}$*.*

Proof. In each of these games, given prices $s_i = \frac{a}{2}$, $p_i = 0$, and $q_i > a$, suppose that all consumers, except the one located at point θ, subscribe to network i. Then this consumer realizes utility

$$\frac{(a - s_i)^2}{2(b+c)} - \lambda\theta = \frac{a^2}{8(b+c)} - \lambda\theta$$

if she subscribes to network i. Since $q_i > a$, she would make no outgoing calls and therefore realize non-positive utility if she subscribed to network j. Therefore, for $\frac{a^2}{8(b+c)} \geq \lambda$, the consumer at θ joins network i for every $\theta \in [0,1]$. Under this condition, network j makes zero profit for any (s_2, p_2, q_2), and network i maximizes its profit by setting $s_i = \frac{a}{2}$. This establishes $n_i = 1, n_j = 0$ as an equilibrium. □

The intuition of the proof is as follows. When the quantity and subscription choices are simultaneous, a "large" network can set a high termination fee to reduce the number of phone calls that reach it (originating from the other network). Then a customer of the other network is essentially restricted to calls within her (small) network and will realize low utility. Thus, such actions of a large network will result in more consumers leaving the smaller network and joining the larger one. The small network is unable to effectively counter the high termination fee of the larger network, until, at equilibrium, the "small" network has no subscribers. Therefore, in this case, corner equilibria always exist.[17]

Proposition 7 indicates that, when switching costs are low, reciprocity does not eliminate the corner equilibria if preference for variety is weak. This is in contrast with the results of proposition 6. Therefore imposing reciprocity may not be as effective if consumers have low switching costs.

6 Concluding Remarks

We have analyzed the effects of strategic commitments in interconnection fees in two-way networks. We find that commitment in interconnection fees by a dominant network implies a price squeeze of the other network. If networks choose interconnection fees that obey the principle of reciprocity (i.e., the networks charge each other equal amounts for call termination), the strategic advantage of the first mover is eliminated, and prices of end-to-end services are lower. Further, under the constraint of reciprocity, the equilibrium termination fees are zero and imputation holds, so that each network charges itself as much as it charges others for the same service. The imposition of reciprocity internalizes the vertical externality, eliminates the "double marginalization," and results in lower prices even in comparison to the simultaneous-action pricing game. Under reciprocity, both consumers' surplus and industry profits are higher than in the simultaneous pricing game. This suggests the value of requiring reciprocity in setting interconnection charges.

The subscription decision stage typically has multiple equilibria, including corner ones, where all consumers subscribe to only one network. However, when reciprocity is imposed, the network with the strategic advantage chooses to set termination fees at cost. As a result, there are no corner equilibria. This is an added benefit of reciprocity, since a corner equilibrium would result in a significant "transportation cost" welfare loss.

The main part of our analysis is done in game structures consistent with the assumption

[17]This is in contrast to the results of Laffont, *et al.* [14] who find that no corner equilibria exist in the case of low switching costs. The difference arises from the fact that they *assume* that the termination fee is low and it is exogenously given.

that the subscription decision is less flexible than both pricing decisions by the firms and quantity choices of consumers. We believe that this assumption is currently seen by telecommunications providers as realistic. However, in the extensions section we explore the possibility of allowing subscribers to simultaneously choose network affiliation and consumption levels, after networks have chosen prices. We point out that in such a setup, there are consumers' coordination problems that make it difficult to even write the maximization problem of the firms without specific assumptions regarding the way they coordinate.

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Appendix

Proof of Proposition 1

In the separable case, the first order conditions are

$$\begin{aligned} x(s_i) + s_i x'(s_i) &= 0, \\ x(p_i + q_j) + p_i x'(p_i + q_j) &= 0, \\ x(p_j + q_i) + q_i x'(p_j + q_i) &= 0. \end{aligned}$$

The first equation implies $s_1 = s_2 = s^m$. Subtracting the third from the second equation yields $(p_i - q_i)\, x'(p_i + q_j) = 0$, which implies $p_i = q_i = \frac{t^o}{2}$, where t is defined as $x(t^o) + \frac{t^o}{2} x'(t^o) \equiv 0$.

With non-separable preferences, maximizing Π_i with respect to s_i, p_i, and q_i, given s_j, p_j, and q_j yields the equilibrium prices. □

Proof of Proposition 2

The five numbers indicated in the proposition are:

$$\lambda_1^{(1)} \equiv \frac{5a^2 b}{18(c+2b)^2}, \quad \lambda_2^{(1)} \equiv \frac{5a^2}{72(b+c)}, \quad \lambda_3^{(1)} \equiv \frac{a^2}{8(b+c)}, \quad \lambda_4^{(1)} \equiv \frac{a^2(9c+13b)}{36(b+c)(2b+c)}, \quad \lambda_5^{(1)} \equiv \frac{13a^2}{72b}.$$

With $c = 0$, the preferences are separable, and

$$\begin{aligned} \lambda_1^{(1)} &= \lambda_2^{(1)} = \frac{5a^2}{72b} = v(s^m) - v(t^o), \\ \lambda_3^{(1)} &= \frac{a^2}{8b} = v(s^m), \\ \lambda_4^{(1)} &= \lambda_5^{(1)} = \frac{13a^2}{72b} = v(s^m) + v(t^o). \end{aligned}$$

First, $\{0,0\}$ is always an equilibrium: each consumer has no incentive to subscribe to any network if no other consumer subscribes.

Second, the corners $\{0,1\}$ and $\{1,0\}$ are equilibrium outcomes if and only if

$$V^{(1)}(1,0) - \lambda \geq \max\left\{0, V^{(1)}(0,1)\right\}.$$

In the non-separable case, this is equivalent (for $n_i = 1 - n_j = 1$) to

$$\frac{a^2}{8(b+c)} - \lambda \geq \max\left\{0, \frac{(a - p_j - q_i)^2}{2(b+c)}\right\}.$$

Since the equilibrium prices p_j and q_i are arbitrary, corner equilibria exist for any $\lambda \leq \lambda_3^{(1)}$. In the separable case, the inequality above becomes

$$v(s^m) - \lambda \geq \max\{0, v(p_j + q_i)\}$$

which is satisfied for arbitrary prices p_j and q_i if and only if $\lambda \leq v(s^m)$.

Any other pair $\{n_1, n_2\}$, such that $n_1 + n_2 \leq 1$, is an equilibrium if and only if

$$V(n_i, n_j) - \lambda n_i = \max\{0, V(n_j, n_i) - \lambda(1 - n_i)\}, \quad i = 1, 2. \tag{3}$$

In words, the marginal consumer subscribing to network 1 (resp. 2), located at point n_1 (resp. $1 - n_2$), must earn the same payoff as his next best alternative. This condition is necessary because, if it does not hold, the consumers located in some neighborhood of n_1, or $1 - n_2$, have an incentive to revise their subscription decision. The condition is also sufficient because it implies

$$V(n_i, n_j) - \lambda d > \max\{0, V(n_j, n_i) - \lambda(1 - d)\}, \text{ for all } d \in [0, n_i);$$

thus no inframarginal consumer of any network has any incentive to revise her subscription decision.

In principle, condition (3) can be satisfied in four cases, considering all possible combinations of equalities and inequalities. However, $V(n_i, n_j) - \lambda n_i = 0$ implies

$$\begin{aligned} V(n_j, n_i) - \lambda n_j &= \max\{0, V(n_i, n_j) - \lambda(1 - n_j)\} \\ &= \max\{0, \lambda n_i - \lambda(1 - n_j)\} \\ &= \max\{0, \lambda(n_i + n_j - 1)\} \\ &= 0. \end{aligned}$$

Thus only two cases are possible: that is, either

$$V(n_i, n_j) - \lambda n_i = V(n_j, n_i) - \lambda(1 - n_i) \geq 0, \quad i = 1, 2; \tag{4}$$

or

$$V(n_i, n_j) - \lambda n_i = 0 > V(n_j, n_i) - \lambda(1 - n_i), \quad i = 1, 2. \tag{5}$$

First, suppose that (4) holds. Then, summing the two equalities and simplifying yields $n_1 + n_2 = 1$, i.e., the two networks cover the whole market. Thus the equalities in (4) are equivalent to the single equation in τ

$$V(\tau, 1 - \tau) - V(1 - \tau, \tau) = \lambda(2\tau - 1). \tag{6}$$

In the non-separable case, this equation has solutions $\tau_1 = \frac{1}{2}$, $\tau_2 = \frac{1}{2} - \delta(\lambda)$, and $\tau_3 = \frac{1}{2} + \delta(\lambda)$. The pair $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ is an equilibrium if and only if

$$V\left(\frac{1}{2}, \frac{1}{2}\right) - \frac{1}{2}\lambda \geq 0,$$

or, equivalently, $\lambda \leq \lambda_3$.

Since δ is increasing in λ, $\delta\left(\lambda_1^{(1)}\right) = 0$ and $\delta\left(\lambda_2^{(1)}\right) = \frac{1}{2}$, the pairs $\left\{\frac{1}{2} + \delta(\lambda), 1 - \delta(\lambda)\right\}$ and $\left\{\frac{1}{2} - \delta(\lambda), 1 + \delta(\lambda)\right\}$ can be equilibrium outcomes only if $\lambda_1^{(1)} \leq \lambda \leq \lambda_2^{(1)}$. This last condition is also sufficient, since it implies that the inequalities in (4) are satisfied.

In the separable case, equation 6 becomes linear in τ. For $\lambda \neq v(s^m) - v(t^o)$, the only solution is $\tau_1 = \frac{1}{2}$. Thus $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ is an equilibrium if and only if

$$V\left(\frac{1}{2}, \frac{1}{2}\right) - \lambda\frac{1}{2} \geq 0$$

i.e., $0 < \lambda \leq v(s^m) + v(t^o)$.

For $\lambda = v(s^m) - v(t^o)$, $\{n, 1-n\}$ is an equilibrium for any $n \in [0,1]$, since $V(n, 1-n) - (v(s^m) - v(t^o))\, n = v(t^o) > 0$.

Now suppose that (5) holds. Then, all solutions different from $\{0,0\}$ must have both n_1 and n_2 strictly positive, because, in the non-separable case, $n_i = 0$ and $V(n_i, n_j) - \lambda n_i = 0$ imply $\frac{a^2(4bn_j)}{72b(b+cn_j)} = 0$, i.e., $n_j = 0$; and in the separable case, $n_i = 0$ and $n_i v(s^m) + n_j v(t^o) - \lambda n_i = 0$ imply $n_j = 0$.

In the separable case, subtracting one equality in (5) from the other yields:

$$(n_i - n_j)(v(s^m) - v(t^o)) = \lambda(n_i - n_j),$$

which implies $n_i = n_j$ unless $\lambda = v(s^m) - v(t^o)$. For $\lambda \neq v(s^m) - v(t^o)$, $n(v(s^m) + v(t^o) - \lambda) = 0$ implies $n = 0$; hence no other equilibrium exists. For $\lambda = v(s^m) - v(t^o)$, the equality $n(v(s^m) + v(t^o) - \lambda) = 0$ holds for any n; hence $\{n, n\}$ is an equilibrium for any $n \in \left[0, \frac{1}{2}\right]$.

In the non-separable case, rewriting the equalities in (5) as

$$a^2\left(4bn_j + 9bn_i + 9cn_in_j + 9cn_i^2\right) = \lambda n_i 72(b + cn_i + cn_j)(b + cn_i)$$

dividing through by n_i, subtracting one from the other and rearranging yields

$$-\frac{4a^2b}{n_1n_2}(n_1 + n_2)(n_1 - n_2) = 72\lambda c(b + c(n_1 + n_2))(n_1 - n_2),$$

which implies $n_1 = n_2$: in fact, if $n_1 - n_2 \neq 0$, then dividing by $(n_1 - n_2)$ yields

$$-\frac{4a^2 b}{n_1 n_2}(n_1 + n_2) = 72\lambda c(b + c(n_1 + n_2)),$$

a contradiction.

Thus, the two equalities in (5) are equivalent to $0 = V(n, n) - \lambda n$, or

$$\frac{1}{n} V(n, n) = \lambda. \tag{7}$$

If $n \in \left[0, \frac{1}{2}\right]$ satisfies (7) then $\{n, n\}$ is an equilibrium, since it also satisfies the inequalities in (5):

$$V(n, n) - \lambda(1 - n) \;=\; \lambda n - \lambda(1 - n) \leq 0$$

Since $\frac{1}{n} V(n, n)$ is decreasing in n and $2V\left(\frac{1}{2}, \frac{1}{2}\right) = \lambda_3$, there is no equilibrium where (5) holds if $0 < \lambda < \lambda_3^{(1)}$. For any λ such that $\lambda_3^{(1)} \leq \lambda < \lambda_4^{(1)}$, there is a unique n that satisfies (7), given by $n_*^{(1)}(\lambda)$. □

Proof of Proposition 4

Substituting the equilibrium prices into the consumers utility functions yields

$$V_{(2)}^1(n_1, n_2) = \frac{a^2}{72b}(4n_2 + 9n_1)$$

and

$$V_{(2)}^2(n_2, n_1) = \frac{a^2}{32b}(n_1 + 4n_2).$$

Proceeding as in the proof of proposition 2, $\{1, 0\}$ is an equilibrium for $\lambda \leq \frac{5a^2}{72b}$ and $\{0, 1\}$ is an equilibrium for $\lambda \leq \frac{3a^2}{32b}$.

For all other pairs $\{n_1, n_2\}$, we can restrict attention to the two cases:

$$V_{(2)}^i(n_i, n_j) - \lambda n_i \;=\; V_{(2)}^i(n_j, n_i) - \lambda(1 - n_i) \;>\; 0, \quad i = 1, 2,$$

and

$$V_{(2)}^i(n_i, n_j) - \lambda n_i = \; 0 \;>\; V_{(2)}^i(n_j, n_i) - \lambda(1 - n_i), \quad i = 1, 2,$$

As in game 1, the first case implies full coverage, $n_1 + n_2 = 1$; and the equality for the marginal consumer is:

$$\frac{a^2}{18b} + \frac{5a^2}{72b} n - \lambda n = \frac{a^2}{32b} + \frac{3a^2}{32b}(1 - n) - \lambda(1 - n)$$

with solution $n = n^{(2)}(\lambda)$. The pair $\left\{n^{(2)}(\lambda), 1 - n^{(2)}(\lambda)\right\}$ is an equilibrium for $\lambda \leq \frac{5a^2}{72b}$, since this implies

$$V^{i}_{(2)}\left(n^{(2)}(\lambda), 1 - n^{(2)}(\lambda)\right) - \lambda n^{(2)}(\lambda) \geq 0.$$

In the other case, the system

$$\begin{aligned} V^{1}_{(2)}(n_1, n_2) &= \frac{a^2}{72b}(4n_2 + 9n_1) - \lambda n_1 = 0 \\ V^{2}_{(2)}(n_2, n_1) &= \frac{a^2}{32b}(n_1 + 4n_2) - \lambda n_2 = 0 \end{aligned}$$

yields $n_1 = n_2 = 0$. □

Proof of Proposition 6

Corner equilibria do not exist since $V^{(3)}(1) = \frac{a^2}{8(b+c)} > 0$ and, for any $\lambda > 0$,

$$V^{(3)}(1) - \lambda < \max\left\{0, V^{(3)}(1)\right\}.$$

Any other point $\{n_1, n_2\}$, with $n_1 + n_2 \leq 1$, is an equilibrium if and only if

$$V^{(3)}_i(n_i, n_j) - \lambda n_i = \max\left\{0, V^{(3)}_j(n_j, n_i) - \lambda(1 - n_i)\right\} \quad i = 1, 2.$$

As in the proof of proposition of game 1, only two cases are possible. Case 1 implies full coverage; hence $\{n, 1 - n\}$ is an equilibrium only if

$$\frac{a^2}{8(b+c)} - \lambda n = \frac{a^2}{8(b+c)} - \lambda(1 - n)$$

which implies $n = \frac{1}{2}$. If $\lambda \leq \frac{a^2}{4(b+c)}$, then $V^{(3)}_i\left(\frac{1}{2}, \frac{1}{2}\right) - \lambda\frac{1}{2} \geq 0$. Thus $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ is an equilibrium if and only if $\lambda \leq \frac{a^2}{4(b+c)}$.

In case 2, $V^{(3)}_i(n_i, n_j) - \lambda n_i = \frac{a^2(n_i + n_j)}{8(b + c(n_i + n_j))} - \lambda n_i = 0$ implies

$$a^2(n_i + n_j) - \lambda n_i 8(b + c(n_i + n_j)) = 0, \quad i = 1, 2.$$

Subtracting one equality from the other yields

$$\lambda 8(b + c(n_1 + n_2))(n_1 - n_2) = 0,$$

which implies $n_1 = n_2$: otherwise, dividing by $(n_1 - n_2)$ generates a contradiction. Thus the two equalities are equivalent to

$$V^{(3)}_i(n, n) - \lambda n = \frac{a^2 n}{4(b + 2cn)} - \lambda n = 0,$$

which implies $n = 0$ or $n = n_*^{(3)}(\lambda)$. Since $n_*^{(3)}(\lambda)$ is decreasing in λ, $n_*^{(3)}\left(\frac{a^2}{4(b+c)}\right) = \frac{1}{2}$ and $n_*^{(3)}\left(\frac{a^2}{4b}\right) = 0$, $\left\{n_*^{(3)}(\lambda), n_*^{(3)}(\lambda)\right\}$ is an equilibrium for $\lambda \in \left[\frac{a^2}{4(b+c)}, \frac{a^2}{4b}\right]$.

In the separable case, case 1 implies

$$v(t^m) - \lambda n = v(t^m) - \lambda(1-n),$$

i.e., $n = \frac{1}{2}$. Thus $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ is an equilibrium for all λ such that $v(t^m) - \frac{1}{2}\lambda \geq 0$, i.e., $\lambda \leq 2v(t^m)$.

In case 2, $(n_i + n_j) v(t^m) - \lambda n_i = 0$, for $i = 1, 2$, implies $n_i = n_2 = 0$. □