Zero Lower Bound on Inflation Expectations*

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Abstract

We document a new fact: in U.S., European and Japanese surveys, households do not expect deflation, even in environments where persistent deflation is a strong possibility. This fact stands in contrast to the standard macroeconomic models with rational expectations. We extend a New Keynesian model with a zero-lower bound on inflation expectations. Unconventional monetary policies, such as forward guidance, are weaker. In liquidity traps, the government spending output multiplier is finite, and adverse aggregate supply shocks are not expansionary. A confidence-driven liquidity trap steady state with deflation does not exist.

Keywords: inflation expectations, non-rational beliefs, survey data
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1 Introduction

The Great Depression has left many enduring marks not only on millions of people but also on macroeconomic thinking and policy. Among many subsequent changes such as launching the Keynesian framework and active countercyclical policy, the depression instilled a fear of deflation. Indeed, generations of central bankers have been concerned about avoiding deflationary spirals and developing deflationary expectations, even if at times deflation appeared to be a remote possibility.[1] The experience of Japan in the 1990s reinforced the view that deflations can be highly damaging and thus should be avoided at any cost. Consistent with this view, central banks utilized everything at their disposal—including unconventional policies such as quantitative easing, forward guidance, and negative nominal interest rates—to escape the specter of deflation during the Great Recession and the COVID-19 crisis. These policy responses have strong support in theoretical models where economic agents have full information rational expectation (FIRE). But given mounting evidence that households and firms exhibit significant departures from FIRE, should one be concerned about deflation as much as predicted by the mainstream view?

We document that, unlike professional forecasters (and financial markets), households—and likely firms, although the evidence for firms is scarce given the dearth of high-quality surveys of firms' inflation expectations—see little chance of a deflation even when actual inflation significantly dips into the negative territory. Specifically, survey evidence shows that inflation expectations are highly asymmetric. In countries with histories of stable and low inflation, households often predict inflation to be well above targets (e.g., five percent or more for an inflation target of two percent) but they forecast negative inflation very rarely. It is as if there is no ceiling for households' expected inflation but there is a floor at zero. In other words, there could be a zero lower bound (ZLB) for expected inflation! At the same time, households' expectations exhibit no ZLB-like property for other macroeconomic expectations (e.g., expectations for unemployment are relatively symmetric). Using a range of surveys for the U.S. and other economies, we document that this empirical pattern holds broadly across periods and countries thus raising the prospect that deflationary fears are unlikely to apply to households. The lack of such fears can shed new light on a variety of phenomena that could be difficult to explain within FIRE. For example, the lack of deflation (or disinflation) during the Great Recession could be rationalized by the fact that few households anticipated deflation.

While being agnostic about sources of ZLB for expected inflation (which could occur due to, e.g., experience-based inflation expectation, dynamics of salient prices, ambiguity

[1] Although Bernanke (2002) devoted a lecture to discussing the dangers of deflation, he asserted, "the chance of significant deflation in the United States in the foreseeable future is extremely small."

aversion, etc.), we explore the implication of this apparent constraint for macroeconomic dynamics and policy in the standard New Keynesian model. Specifically, we consider scenarios where ZLB can apply to nominal interest rates (to avoid confusion, we refer to this case as the effective lower bound, or ELB), expected inflation or both. With inflation expectations being unable to fall below zero (more generally some cutoff), an ELB on nominal interest rates continues to limit the ability of the central bank to stimulate the economy, but the risk of deflationary spirals could be effectively eliminated. Intuitively, an ELB on nominal interest rates can create preconditions for a deflationary spiral because the central bank cannot respond to shocks. At the same time, a ZLB on expected inflation can obviate this possibility by countering deflationary pressures in the economy: via the Phillips curve, actual inflation is less likely to turn deeply negative if expected inflation is stuck at zero. This result can explain why Japan has not demonstrated elevated macroeconomic volatility predicted by FIRE-based models despite spending nearly three decades at the ELB on nominal interest rates.

The ZLB constraint on expected inflation also affects the effectiveness of unconventional policy tools based on the management of expectations. For instance, the presence of a ZLB on expected inflation weakens the effectiveness of forward guidance, since expected inflation is no longer sensitive to policy announcements. Likewise, persistent negative interest rates are less powerful when inflation expectations are constrained by its lower bound. This is because anticipation of future negative interest rates may not lift inflation expectations above its constraint. Thus, the effect of negative interest rates policy is limited to its contemporaneous effect only. Average inflation targeting (AIT) may also be less attractive because inflationary shocks could be harder to control when households do not anticipate a possibility of deflation in the future (more realistically, significant disinflations).

The ZLB on inflation expectations qualitatively modifies economy's response to shocks. For example, standard New Keynesian models with the FIRE beliefs predict that negative aggregate supply shocks, such as higher taxes on firms, increase output gap during the ELB on the nominal interest rate. This prediction appears to be at odds with the data. Relatedly, according to the standard model, government spending multipliers can be arbitrarily large with the nominal interest rate stuck at the ELB while the empirical evidence points to larger but still relatively modest multipliers. We show that these predictions of FIRE-based models are no longer the case when inflation expectations are at zero.

With the ZLB on expected inflation, the properties of self-fulfilling equilibria change in significant ways. Typical New Keynesian models with Taylor rules that strongly react to inflation unless constrained by the ELB feature two steady states. In a conventional steady state, inflation is at its target, and the output gap equals zero. In a liquidity trap steady state, the nominal interest rate is zero, inflation and output gap are negative. Importantly, inflation expectations are also negative. The ZLB on inflation expectations rules out such a steady state by preventing agents from believing in deflation. Moreover, we show that even temporary confidence-driven liquidity traps are ruled out if the economy does not face fundamental shocks. If confidence and fundamental shocks occur simultaneously, the ZLB on inflation expectations can increase the number of equilibria in the model.

Our analysis contributes to several strands of research. First, our work is related to the large literature examining properties of inflation expectation for households and firms (see Coibion, Gorodnichenko and Kamdar, 2018 for a survey). Generally, this line of work finds that inflation expectations of households in advanced economies have little resemblance to FIRE. We add to the list of departures from FIRE by documenting that households appear to downplay the possibility of future deflation to a point when they think deflations effectively can't happen. The closest paper in this literature is Baqaee (2020) which studies asymmetry in the pass-through of actual inflation into households' inflation expectation. Specifically, he finds that negative inflation shocks have a weaker effect on households' inflation expectations than positive shocks. In contrast, we focus on the zero constraint for inflation expectation. In another closely related paper, Andrade, Gautier and Mengus (2020) argue that the intensive margin for inflation expectations appears to play a limited role in determining aggregate inflation expectations of French households. Instead, the extensive margin (i.e., the share of people expecting positive inflation) explains most of the variation. Building on Andrade, Gautier and Mengus (2020), we note that there is little variation in the share of people predicting deflation and hence inflation expectations effectively have a zero lower bound.

Second, we contribute to the vast literature on the importance of inflation expectations for macroeconomic dynamics. For example, Krugman (1998), Eggertsson and Woodford (2003), Eggertsson (2008) and others argued that raising inflation expectations can materially contribute to ending economic depressions. In contrast to this literature, we suggest that, for many advanced economies with low inflation, it is surprisingly hard to generate deflationary expectations for households. For example, even now, most households in Japan expect inflation. To the extent households' expectations are more important for consumption decisions than those of professional forecasters or financial markets (Coibion, Georgarakos, Gorodnichenko and van Rooij, 2019a; D'Acunto, Hoang, Paloviita and Weber, 2019), our results call for a more nuanced interpretation of recent developments for inflation in Japan in the 1990s and afterwards. We also add to the literature studying inflation dynamics during the Great Recession when inflation did not fall as much as one could have expected given the rise of unemployment thus generating a missing disinflation puzzle (Hall, 2011). A variety of theories has been offered to rationalize this anomaly with some authors (e.g., Coibion and Gorodnichenko, 2015b) suggesting that a temporary increase in households' inflation expectations due to a hike in energy prices saved the U.S. economy from a deflation. Our analysis is related to this reasoning but instead of emphasizing the sensitivity of households' inflation expectations to changes in salient prices (e.g., the price of gasoline), we stress that disinflation pressures were attenuated because few households predicted deflation due to the ZLB in inflation expectations.

Our paper adds to the literature that extends business cycle models with non-FIRE beliefs, such as dispersed information (Lucas, 1972; Angeletos and La'O, 2013), rational inattention (Sims, 2003; Maćkowiak and Wiederholt, 2015), statistical learning (Evans and Honkapohja, 2012; Eusepi and Preston, 2011), cognitive discounting (Gabaix, 2020), extrapolation and diagnostic expectations (Bordalo, Gennaioli, Ma and Shleifer, 2020; Bordalo, Gennaioli, Shleifer and Terry, 2021) to name a few. One application of this literature was to close a disconnect between a tremendous power of forward guidance policy in standard New Keynesian models and a lack of it in reality (Angeletos and Lian, 2018; García-Schmidt and Woodford, 2019; Wiederholt, 2015). The ZLB on inflation expectations in our paper also attenuates the strength of forward guidance and generates several other predictions consistent with empirical evidence.

2 Do People Expect Negative Inflation?

This section documents a new fact that households virtually never expect deflation. We present three pieces of evidence. First, we examine the cross-sectional distribution of point forecasts by households to illustrate that few households predict deflation. Second, we study time series variation in the share of households who predict negative, zero or positive inflation. Third, we explore how variations in actual inflation translate into variations of various percentiles of inflation expectations.

2.1 Expectations across Consumers

As a first step, we present evidence for the households in the Michigan Survey of Consumers (MSC) and the Bank of Japan Opinion Survey. We start with the Michigan Survey of Consumers, which is one of the longest surveys of households' expectations. The top panel of Figure 1 plots the histogram of inflation expectations in gray and realized Consumer Price Index (CPI) inflation rate in red between 1985 and 2008. There are two notable observations here. First, the distribution of inflation expectations is highly asymmetric around its mean of 3.7 percent with the right tale stretching to more than 20 percent and the left tale being essentially truncated at zero. Second, while there is almost no one who expected deflation during this period, a significant share of people did expect inflation to be lower than the lowest realized inflation of one percent. Hence, people are capable of expecting inflation which is lower than actual inflation.

With the onset of the Great Recession, the United States witnessed CPI deflation for the first time since the 1950s. The bottom panel of Figure 1 redraws the inflation expectations and realized inflation histogram for the period between 2008 and 2019. The realized inflation histogram shifts to the left crossing the zero mark. At the same time, the distribution of inflation expectations stays to the right of the zero border. It is as if there is a zero lower bound on inflation expectations. This is a central observation of this paper. Is this pattern specific to the U.S.?

The Japanese experience during the last three decades presents an important window into a setting where deflation is a recurrent, persistent phenomenon. Half of all quarters between 1994 and 2021 were in deflation and yet almost no one in Bank of Japan's Opinion Survey, which collects numerical inflation expectations from Japanese consumers, expects deflation.[2] Figure 2 presents a histogram similar to the one plotted for the MSC in Figure 1. Strikingly, the fraction of people who expected deflation between the first quarter of 2004 and the first quarter of 2020 is only five percent, while the actual inflation was in the negative territory approximately 40 percent of the time.[3] Hence, even in a country with a prominent history of deflation, households appear to avoid predicting deflation.

2.2 Expectations over Time

Cross-sectional distributions reported in the previous section are informative, but they do not convey potentially important time-series information. To explore this dimension of the data, we next show time series for the fraction of consumers who expect positive, zero, and negative inflation. We focus on two prominent deflationary episodes of Japan during the last three decades and Greece during the European debt crisis.

Figure 3 displays inflation and core inflation (the top panel) and the evolution of cross-section inflation expectations distribution (the bottom panel) in Japan. The key observation is again that only a modest share of consumers expect deflation. Most of the time series variation in the cross-sectional distribution occurs in the shares of people who expect positive and zero inflation. The fraction of people who expect deflation barely moves.

[2]Bank of Japan's Opinion Survey asks "By what percent do you think prices will change one year from now? Please choose "up" or "down" and fill in the box below with a specific figure. If you think that they will be unchanged, please put a "0." Prices will go up/down about ___ percent one year from now."

[3]There is an atom at zero in the distribution of expected inflation. To make the realized inflation histogram comparable to expected inflation histogram, we treated value of realized inflation in the interval [-0.2%;0.2%] as zero. As a result, realized inflation was negative more than 40 percent of time during this period.

The only exception is the episode around the global financial crisis where many different events unfolded that could have influenced inflation expectations, such as changes in salient prices (Yen's appreciation and a decline in energy and food prices).

Following the European debt crisis, Greece experienced a devastating recession with unemployment rate reaching almost 30 percent in 2013. This deep recession was associated with deflation between 2012 and 2016 as shown in Figure 4. Yet the fraction of people who expected deflation barely moved, staying below 10 percent at all times. This stands in contrast to the behavior of the fraction of people who expected zero inflation. It started rising rapidly after 2010 reaching its peak of more than 40 percent in 2015.

2.3 Revisions of Inflation Expectations

In the presence of the ZLB on inflation expectations, the households at the ZLB constraint should not be able to revise their inflation beliefs downward. In contrast, those households with positive initial inflation expectations should be free to update their beliefs downward as long as they do not hit the ZLB constraint. To test this prediction, we use the Michigan Survey of Consumers data from 1978M1 to 2019M12.[4] Figure 5 plots five histograms of inflation expectations revisions conditional on the initial inflation expectations of one of the five values: $0, 1, 2, 3$ and 5 percent. The key observation in this figure is that revisions typically do not cross a negative of the initial inflation expectations. For example, people who initially predicted zero inflation tend to continue to predict zero inflation or increase their expectations to one percent and above (i.e., a revision between 0 and 10 percent). By contrast, people who initially predicted inflation to be one percent can cut inflation expectations by one percent, continue to predict one-percent inflation, or increase inflation expectations to two percent and above (i.e., a revision between -1 and 10 percent). The same pattern repeats for those with initial inflation expectations of 2, 3, and 5 percent. In short, the behavior of revisions is also consistent with ZLB for inflation expectations.

2.4 Conditional Expectations

Our results so far focus on the central tendencies in the data. While informative, one may be interested in the comovement of inflation expectations and actual inflation at various percentiles. Specifically, we want to examine whether the properties of bottom

[4]Most of the Michigan Survey of Consumers participants are interviewed twice with a 6-month delay. In both interviews, the inflation forecast question asks about inflation over the next 12 months. As a result, we define a revision of inflation forecast as $\pi^e_{t,t+12,i} - \pi^e_{t-6,t+6,i}$, where t is a month index, and i is a household index.

percentiles, which are close to zero, are different from the properties of top percentiles, which are unlikely to be constrained by zero. To this end, we estimate a series of quantile regressions

$$\pi^e_{t,t+12,i} = \alpha^{(\tau)} + \beta^{(\tau)}\pi_{t-12,t} + \gamma^{(\tau)}\mathbf{X}_{i,t} + \varepsilon_{i,t}, \tag{1}$$

where i, t, and τ index respondents, time (month), and percentiles, $\pi^e_{t,t+12,i}$ is the one-year-ahead inflation forecast of household i at time t, $\pi_{t-12,t}$ is the actual inflation rate over the previous 12 month, $\mathbf{X}_{i,t}$ is a vector of respondent characteristics. Because the key variable $\pi_{t-12,t}$ does not vary across respondents, we cluster standard errors by time. We use the Michigan Survey of Consumers data from 1978M1 to 2019M12 and report results for $\tau = \{10, 20, \ldots, 90\}$ in Figure 6.

Consistent with evidence presented earlier, we find that as we consider lower quantiles the sensitivity of expected inflation to actual inflation declines to zero. For example, consider the 10th percentile. The slope estimate is zero thus suggesting that expectations exhibit no reaction to variations in actual inflation. Furthermore, the intercept is also effectively estimated at zero meaning that inflation expectations are set at zero. Note that standard errors for the 10th percentiles are essentially zero. This result indicates that there is little variation in the left tail of the distribution for expected inflation because expectations are bounded by zero. As we raise the percentile, both the slope and the intercept increase. For example, the estimated slope for the median is close to one, i.e., expected and actual inflation co-move roughly by the same magnitude. However, the intercept is close to 1.5 percentage points so that the forecast is systematically biased upwards. The top percentiles are characterized by high sensitivity of expectations to actual inflation (the slope is considerably greater than one) and a large upward bias (the intercept is more than five percentage points).

If actual inflation declines, as it did during the Volcker disinflation, then inflation expectations are pushed close to zero but expectations do not appear to cross the zero bound. Instead, expectations appear to pile at zero or just above zero. In contrast, the right tail of the distribution comoves strongly with actual inflation, i.e., consistent with the simple histograms reported in section 2.1, households' inflation expectations have a floor at zero but have no ceiling. As a result, the average expected inflation is driven by a mix of respondents reporting zero expected inflation and respondents who predict a positive inflation rate. This pattern is also consistent with Andrade, Gautier and Mengus (2020) arguing that variation in the average expected inflation for French households is largely due to variation in the share of people expecting positive inflation, i.e., the extensive margin. Our findings are also consistent with Baqaee (2020). As the Volcker disinflation achieved lower levels of inflation, negative inflation shocks are unlikely to generate reductions in expected inflation because respondents are likely to report zero expected inflation rather than a negative magnitude thus yielding asymmetric responses documented in Baqaee (2020).

2.5 Subjective Uncertainty

Although consumers' point predictions for future price changes suggest that deflation as an unlikely outcome, consumers may still assign positive probability to deflation. The Survey of Consumer Expectations (SCE), run by the Federal Reserve Bank of New York, asks households to report their subjective probability distributions for future inflation. Specifically, households assign probabilities to ten bins spanning the following options: "inflation will be more than 12%", "inflation will be between 8% and 12%", ..., "inflation will be between 0% and 2%", "deflation will be between -2% and 0%",..., "deflation will be more than -12%". Panel A of Figure 7 shows that households put a positive probability on deflation (on average, approx. 16%) but the distribution is skewed towards high-inflation bins. Specifically, households believe that a double-digit inflation (which the U.S. economy has not experienced since the early 1980s) is more likely than deflation (which the U.S. economy experienced during the Great Recession). To better understand the properties of subjective distributions, Panel B of Figure 7 shows the time series for the average probability assigned to each bin. We observe that the variation for deflationary bins is compressed relative to the inflationary bins.[5] This pattern suggests that variation in expected inflation can be largely driven by changes in the probabilities assigned to bins associated with positive inflation. To illustrate this point more clearly, Panel C of Figure 7 plots the time series for the average expected inflation as well as contributions due to inflation and due to deflation. We find that the expected inflation tracks the contribution due to inflation. We conclude that while households put a positive probability on deflation in response to the SCE survey question, the time-series variation in this probability is rather modest.

Why is there so little time-series variation in the probability of deflation as perceived by households? First, we may fail to see discernible variation because actual inflation does not move much and specifically deflation did not happen in the sample period. This is certainly a possibility but CPI inflation did go negative briefly in 2015. Second, the survey question treats inflation and deflation symmetrically. If respondents are not sophisticated/informed, they may assign positive probabilities to outcomes close to the middle of the presented options because they may assume that the middle reflects reasonable outcomes. This priming of responses may result in a pattern where some households can report a positive probability of deflation even if they do not think deflation is

[5] For example, the standard deviation is 0.99 for the "deflation will be between -2% and 0%" bin and and 2.5 for the "inflation will be between 0% and 2%" bin.

likely (Coibion *et al.*, 2021). In other words, we may face a measurement error in survey responses. To gauge the plausibility of this scenario, we can use survey data from the Dutch National Bank. Unlike other surveys of inflation expectations, the Dutch survey of consumers ask respondents to report a *minimum* expected inflation.[6] One can think of this question as asking respondents to predict the worst-case scenario for deflation. Figure 8 shows that only a tiny share of respondents predicts a deflation. This pattern of responses is consistent with potential priming of responses in the SCE-type questions and again highlights a special role of a zero cutoff for inflation expectations.

2.6 Other Variables and Types of Agents

To explore whether ZLB on expected inflation applies to other types of agents, we examine inflation forecasts made by firms and professional forecasters. One may think that these agents have stronger incentives to pay attention to macroeconomic developments and thus generate forecasts with properties that more closely resemble FIRE. After all, professional forecasts make a living by producing high-quality projections and firms should use high-quality projections to set prices. Consistent with this view, we find (the top left panel of Figure 9) that the distribution of one-year-ahead inflation forecasts in the U.S. Survey of Professional Forecasters not only mimics the distribution of actual inflation but also shows few signs of asymmetry. In other words, professional forecasters do not assign any special role to zero when they prepare their inflation forecasts. Using data from Consensus Economics, we find similar evidence for other countries. For example, the top right panel of Figure 9 shows that in Japan, the distribution of inflation expectations by professional forecasters is close to the actual distribution of inflation. This result suggests that the apparent unwillingness of households to predict deflation does not likely reflect a fundamental property of underlying data generating processes.

Although professional forecasters seem to behave in line with FIRE, prior work (e.g., Candia, Coibion and Gorodnichenko, 2021) suggests that managers' inflation expectations could be close to households' inflation expectations. For example, like households' inflation expectations, managers' inflation expectation have large dispersion, upward biases, large and frequent revisions, strong correlation of short- and long-run forecasts, etc. Consistent with the earlier evidence, we find (the bottom left panel of Figure 9) for the U.S. that virtually no manager in the Survey of Firm Inflation Expectations (SoFIE) predicts deflation in the third quarter of 2020 when the realized inflation was the lowest

[6] The DNB Household Survey questionnaire provides the following English translation of the actual question: "What will be the minimum percentage prices could increase over the next twelve months, do you think? If you think prices will decrease, you can fill in a negative percentage by using a minus in front of the number."

during the COVID recession. Furthermore, the bottom right panel of Figure 9 plots the histogram of inflation expectations by Japanese enterprises in the TANKAN survey between 2014Q1 and 2021Q3. The survey asks the participants to pick inflation expectations from the ten values $\{-3, -2, \ldots, 5, 6\}$, rather than allowing to choose continuous values. Nevertheless the distribution of the answers is right-skewed with a sharp drop below zero. Given the short time series dimension of the SoFIE, round number answer options in tha TANKAN survery, and more generally the dearth of high-quality surveys of managers' inflation expectations, this evidence is obviously tentative but it suggests that the ZLB on expected inflation can apply to firms as well.

When we study properties of forecasts for other macroeconomic variables reported by households in the Michigan Survey of Consumers, we observe that households have little trouble predicting negative values. For example, the Michigan Survey of Consumers asks households to report whether they anticipate increase/no change/decrease in unemployment and nominal interest rates. Unfortunately, the questions are qualitative but we can compare shares of people predicting decreases and increases to see if the distribution of responses is symmetric. We find (Figure 10) that these distributions are much more symmetric (especially for unemployment) than the distribution for inflation forecasts. This pattern suggests that households generally can report negative values and the ZLB property for expectation is constrained to price changes.

2.7 Potential Sources of ZLB on Expected Inflation

While expectations are often thought to be flexible variables that can take any value commanded by the economic outlook, our findings suggest that households are highly reluctant to report expected inflation below zero. We observe this pattern for a variety of countries and periods thus suggesting that this phenomenon may be more than a statistical fluke. But why would households hesitate to predict negative inflation? In the absence of exogenous variation in expectations targeted to push households into thinking about possible deflation, one may be unable to provide a beyond-reasonable-doubt account for this empirical regularity. However, we can suggest some tentative hypotheses that can rationalize the observed behavior of inflation expectations.

First, even in countries with low and stable inflation, many households lived through a period of high inflation in the past. For example, although U.S. households have not seen double-digit inflation for nearly 40 years, some households may still remember the Great Inflation of the 1970s and thus may find it hard to believe that inflation can in fact be negative. Malmendier and Nagel (2016) document evidence consistent with this hypothesis. In a similar spirit, German households may be scarred by memories and consequences of the hyperinflation after World War I so that they can be unlikely to report a negative inflation rate. In this case, one does not have to actually live through a hyperinflation but the society should have enough memory collectively to make households believe that the distribution of inflation is asymmetric with high inflation outcomes being much more likely than deflation outcomes. Relatedly, consumers who have not lived through a deflation may simply not understand what deflation (negative inflation) is.

Second, inflation is a complex phenomenon and understanding sources and implications of inflation shocks may be challenging for households who often have limited economics knowledge and financial literacy (D'Acunto *et al.*, 2019). As a result, households may interpret the inflation dynamics through lenses of worst-case scenarios where households perceive inflation as a bad state (Kamdar, 2018). For example, Baqaee (2020) uses ambiguity aversion to rationalize downward rigidity of inflation expectations in response to changes in actual inflation. Relatedly, Afrouzi and Veldkamp (2019) argue that uncertainty about parameters of the data generating process that has a skewed distribution—a likely case for actual inflation—can also generate an upward bias in reported inflation expectations via Jensen's inequality thus reducing a chance of forecasting deflation. This line of reasoning is consistent with the fact that the right tail (high inflation) in the densities reported by households tends to be heavier than the left tail (deflation) and the mean expectation is greater than the median (see e.g., De Bruin, Manski, Topa and Van Der Klaauw, 2011, Coibion, Gorodnichenko and Weber, 2019b).

Third, in low inflation environments, households appear to rely on salient prices for frequently-purchased goods such as gasoline or food to form their inflation expectations (e.g., Coibion and Gorodnichenko, 2015b, Cavallo, Cruces and Perez-Truglia, 2017, D'Acunto, Malmendier, Ospina and Weber, 2021) as these prices provide an easy way to gauge inflation when incentives for track inflation are weak. To the extent inflation for these goods is less likely to experience deflation than the whole CPI, households may conclude that the aggregate level of prices is unlikely to fall. Consistent with this view, the inflation rate for food and beverages in the U.S. during the Great Recession barely moved into a negative territory while the full-basket CPI declined by two percentage points. In a similar spirit, while inflation for the full-basket CPI approached zero during the COVID crisis, the food inflation rose to four percentage points. Obviously, prices for other goods may be salient for other countries. For example, the exchange rate may be more relevant for small open economies or countries with recent histories of high inflation (e.g., Coibion and Gorodnichenko, 2015a). Because food and energy prices may be determined by the global factors or forces that are specific to food and energy, inflation expectations may move for reasons other than domestic business cycles.

Establishing which of these hypotheses—and there could be alternative accounts—is an empirically relevant explanation is beyond the scope of the paper but we would like to note that, while these hypotheses suggest departures from FIRE, they may still be consistent with rationality (e.g., households can economize on information costs to arrive at potentially worse but cheaper expectations). Moreover, the apparent unwillingness of households to predict deflation is unlikely to be structural: U.S. households were expecting deflation in the early 1950s when a different policy regime and stable-price-level mindset were in place (Binder and Brunet, 2021). To make further progress in understanding macroeconomic implications of the ZLB for households' inflation expectations, we will assume that i) households act rationally; ii) for some reason, households believe that future inflation can't be negative; iii) households do not abandon their no-deflation belief in response to shocks. This practical approach allows us to have minimum deviations from the standard New Keynesian model and hence permits analytical tractability that would be very hard to achieve otherwise in a more sophisticated setting for the formation of inflation expectations.

3 ZLB on Inflation Expectations in an NK Model

This section presents a simple representative agent three-equation New Keynesian model with a twist. Inflation expectations is constrained to be positive even when a FIRE agent would predict inflation to be negative. Because the model is standard otherwise, we skip the description of the fundamentals and jump directly to equilibrium conditions, taking particular care of expectations.

In this economy, people may be surprised by unfolding events because their state-dependent non-FIRE forecasts are different from realized outcomes in some states. As a result, we have to separately solve for forecasts and for realized variables. Formally, there are two sets of equilibrium equations: those that determine contemporaneous endogenous variables and those that govern the behavior of forecasts.

Contemporaneous equations. We present a log-linearization of equilibrium conditions around steady state with log inflation $\overline{\pi} > 0$ and output gap of zero. The Euler equation takes the familiar form

$$x_t = \mathbb{E}_t[\widetilde{x}_{t+1}] - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t[\widetilde{\pi}_{t+1}] - r_t^n\right), \tag{2}$$

where r_t^n is the exogenous natural interest rate with the steady state value of $\overline{r} \equiv -\log\beta$, i_t is the nominal interest rate set by the central and its steady state value is $\iota \equiv \overline{r} + \overline{\pi}$, and π_t is the inflation rate and $\overline{\pi}$ is its steady state value, x_t is an output gap, and a positive parameter $1/\sigma$ is the intertemporal elasticity of substitution. Tilde on top of the endogenous variables highlights that the forecasts do not necessarily coincide with FIRE expectations. In our notation, all endogenous variables are functions of past and current exogenous shocks (see Iovino and Sergeyev (2018) for a similar notation). Thus non-FIRE forecasts that bear tildes, such as $\widetilde{x}_{t+1}$, are functions that differ from the actual mappings from shocks to realizations, such as x_{t+1}.[7] The expectations operator $\mathbb{E}_t[\cdot]$ is mathematical expectations conditional on information at time t over exogenous shocks that have not yet been realized. The agents are perfectly aware of the distribution of future exogenous shocks. We assume that agents do not change their forecasting functions over time. As a result, these functions bear only one time subscript that refer to the period for which the forecast is made.[8]

When firms can perfectly index their prices to the constant trend inflation $\overline{\pi}$, the Phillips curve is

$$\pi_t - \overline{\pi} = \kappa x_t + \sum_{k=1}^{\infty} (\beta\theta)^k \mathbb{E}_t \left[\kappa \widetilde{x}_{t+k} + \frac{1-\theta}{\theta} \left(\widetilde{\pi}_{t+k} - \overline{\pi} \right) \right] \tag{3}$$

where $\beta > 0$ is the subjective discount factor of the representative household, $\kappa > 0$ is the slope of the standard New Keynesian Phillips curve, and $1 - \theta > 0$ is the probability with which firms re-optimize their prices. Appendix A.1 derives the Phillips curve (3). When the trend inflation $\overline{\pi}$ is zero and the expectations are rational, equation (3) collapses to the standard New Keynesian Phillips curve. However, when expectations are not rational, we cannot fold equation (3) into its rational expectations form except for a few convenient special cases.[9]

[7] A popular alternative to our distorted beliefs notation is distorted distributions over realization of endogenous variables. This alternative and our notation are equivalent. To see why, take a two-period example where endogenous variable π is a function of the exogenous and random state r^n in the second period. With our notation, the beliefs are captured by forecasting function $\widetilde{\pi}(r^n)$, which summarizes the forecast in every future state. The mean belief is $\mathbb{E}[\widetilde{\pi}(r^n)] = \int \widetilde{\pi}(r^n) dF(r^n)$, where F is a known cumulative distribution function (cdf) of r^n. Using the alternative representation, one can write $\widetilde{\mathbb{E}}[\pi] = \int \pi d\widetilde{F}(\pi)$, where $\widetilde{F}$ is a distorted cdf of π. The relation between these distribution functions is $d\widetilde{F}(\widetilde{\pi}(r^n)) = dF(r^n)$. For example, the zero lower bound on inflation expectations in our notation is a constraint on forecasting function, i.e., $\widetilde{\pi} \geq 0$. Under the alternative notation, the same constraint can be represented by zero weight attached to negative values of π in the distribution function $\widetilde{F}(\pi)$.

[8] In principle forecasting functions $\widetilde{x}_{t+1}$ can depend on the time the forecast is made, that is, for example, $x_{t,t+1}$. The fact that forecasting functions do not change over time does not rule out a potential Bayesian learning about the exogenous state of the economy. However, this rules out statistical learning, such as constant gain learning, where the agents change the mapping from shocks to forecasts over time.

[9] Adam and Padula (2011) show that log-linearized equilibrium equations of a micro-founded New Keynesian model produce a standard expectations-augmented New Keynesian Phillips curve when agents do not expect predictable movements in their non-FIRE beliefs. We do not obtain the same result despite the fact that the agents in our model also do not expect predictable movements in their expectations. This is because the agents in our model hold beliefs that may not be consistent with equilibrium relations in some

The central bank follows a Taylor rule of the form

$$i_t = \max\left\{0, \iota + \phi\left(\pi_t - \overline{\pi}\right)\right\}, \tag{4}$$

where $\phi > 1$ and the interest rate cannot become negative, which we refer to as the effective lower bound on the nominal interest rate, or the ELB.

Forecasts. The representative agent does not have FIRE. In accordance with our empirical evidence in Section 2, we posit that the agent believes that future inflation cannot fall below zero. Formally, beliefs satisfy the analogues of equations (2)-(4). The Euler equations (2) at some state of future period $t+s$ is

$$\widetilde{x}_{t+s} = \mathbb{E}_{t+s}\left[\widetilde{x}_{t+s+1}\right] - \frac{1}{\sigma}(\widetilde{i}_{t+s} - \mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1}\right] - r^n_{t+s}), \tag{5}$$

for all $s \geq 1$. The key difference between the contemporaneous and future Euler equations (2) and (5) is that the contemporaneous equation has the realized output gap x_t on the left-hand side, while the future equation (5) has a non-FIRE forecast instead.

The analogue of the Philips curve is

$$\widetilde{\pi}_{t+s} = \max\left\{\overline{\pi} + \kappa\sum_{k=0}^{\infty}(\beta\theta)^k\mathbb{E}_{t+s}\left[\widetilde{x}_{t+s+k}\right] + \frac{1-\theta}{\theta}\sum_{k=0}^{\infty}(\beta\theta)^{k+1}\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1+k} - \overline{\pi}\right], 0\right\}, \tag{6}$$

for all $s \geq 1$ and where the max operator introduces a zero-lower bound, or the ZLB, constraint on inflation forecasts. The equation states that whenever the agent expects positive inflation, it equals the first argument of the max operator. Otherwise, the agent expects zero inflation. All endogenous variables bear tilde highlighting that forecasts may deviate from rational expectations. Finally, note that by having the same forecasts in the Euler equation and the Phillips curve, we effectively assume that households and firms have identical beliefs. In particular, this means that both households and firms are subject to the zero lower bound on inflation expectations.

Finally, the Taylor rule is

$$\widetilde{i}_{t+s} = \max\{0, \iota + \phi\left(\widetilde{\pi}_{t+s} - \overline{\pi}\right)\}. \tag{7}$$

Shocks. We consider an uncertainty structure where at the beginning, the economy is hit with a negative shock to natural interest rate $r^n_t = \underline{r} < \overline{r}$ that reverts back to its steady

future periods and states. For example, equation (A2) in Adam and Padula (2011) is in general not true in our model.

state level with probability $\mu \in [0, 1]$, and with remaining probability, it stays at $\underline{r}$. The initial negative natural interest rate shock can trigger either the ELB constraint or the ZLB on inflation expectations constraint or both. The constant probability of reversal to the steady state allows us to solve for Markov equilibria, which we can conveniently visualize with diagrams.[10]

3.1 FIRE Benchmark

A useful starting point is a rational expectations equilibrium that does not feature zero lower bound on inflation expectations. We use backward induction to solve the model. First, after the natural interest shock subsides, the economy immediately returns to its steady state because there are no state variables in this model. We denote a (stochastic) period when this happens as T. Thus we have that $x_t = 0, \pi_t = \overline{\pi}, i_t = \iota$ for all $t \geq T$.

For periods $t < T$, the Euler equation is

$$x_L = -\frac{1}{\mu\sigma}\left[\max\left\{0, \overline{r} + \overline{\pi} + \phi\left(\pi_L - \overline{\pi}\right)\right\} - (1-\mu)\pi_L - \mu\overline{\pi} - \underline{r}\right], \tag{8}$$

where x_L and π_L are output gap and inflation when the exogenous natural real interest rate r_t^n equals $\underline{r}$. Output gap and inflation only depend on the natural real interest rate because there are no endogenous state variables. Figure 11 plots a piece-wise linear Euler equation in the (x_L, π_L) space for several values of r^n. When inflation is sufficiently low, the Taylor rule prescribes the central bank to set a zero interest rate and the output gap-inflation relation turns positive.

The Phillips curve (3) gives

$$\pi_L - \overline{\pi} = \frac{\kappa}{1 - \beta(1-\mu)} x_L. \tag{9}$$

It is represented by upward-sloping line in Figure 11. When a negative shock to natural interest rate is small enough, it does not trigger the ELB on the nominal interest rate. The Euler equation and the Phillips curve jointly determine output gap

$$x_L = \frac{1 - \beta(1-\mu)}{\mu\sigma[1 - \beta(1-\mu)] + (\phi + \mu - 1)\kappa}\left(\underline{r} - \overline{r}\right) < 0, \tag{10}$$

which together with the Phillips curve (9) determine inflation. This solution highlights that output gap and the inflation rate deviations from their steady state values are both

[10] It is straightforward to analytically solve the model in the case of a deterministic duration of shocks to natural real interest rate.

negative, which is also represented by the intersection A_1 of PC and EE' curves in Figure 11.

If a shock to the natural interest rate is sufficiently large, the ELB constraint on the interest rate becomes binding. The curve EE'' and its intersection with the Phillips curve PC in point A_2 in Figure 11 present such a case. The Euler equation and the Phillips curve intersect at

$$x_L = \frac{1-\beta(1-\mu)}{\vartheta(\mu)}\left[\iota + (\underline{r}-\bar{r})\right], \tag{11}$$

where $\vartheta(\mu) \equiv \mu\sigma[1-\beta(1-\mu)] - (1-\mu)\kappa$. The inflation rate is given by (9). For this solution to be bounded and unique, the denominator $\vartheta(\mu)$ in equation (11) has to be positive

$$\vartheta(\mu) = \mu\sigma[1-\beta(1-\mu)] - (1-\mu)\kappa > 0. \tag{12}$$

When the persistence of the negative interest rate shock $1-\mu$ is lower than a certain cutoff, this condition holds. Graphically, condition (12) requires that the upward-sloping part of the Euler equation is steeper than the Phillips curve in Figure 11.

For the ELB to bind, the interest rate $\underline{r}$ must be below the following cutoff

$$\underline{r}_{i,\text{REE}} \equiv -\frac{\mu\sigma[1-\beta(1-\mu)] - \kappa\,(1-\mu)}{\kappa\phi}\bar{r} - \frac{\mu\sigma[1-\beta(1-\mu)] + \kappa\,(\phi+\mu-1)}{\kappa\phi}\overline{\pi}. \tag{13}$$

The first term in condition (13) is the standard one that indicates the minimal size of the natural rate shock that is needed to generate the ELB with zero steady state inflation. The second term shows that the shock must be larger in absolute terms in the presence of a positive steady state inflation.

When firms index their prices to trend inflation, the inflation rate need not be negative in the ELB on the nominal interest rate as illustrated by the intersection A_2 of the Phillips curve PC and the Euler equation EE'' in Figure 11. For inflation to be negative, the natural interest rate needs to fall below the threshold

$$\underline{r}_{\pi,\text{ELB}} \equiv -\mu\left[\frac{1-\beta(1-\mu)}{\kappa}\sigma + 1\right]\overline{\pi}.$$

This situation occurs when the Euler equation curve is shifted to the left sufficiently as depicted by the curve EE''' in Figure 11.

Depending on the parameters of the model, the cutoff value $\underline{r}_{\pi,\text{ELB}}$, where the inflation turns negative, can be below or above $\underline{r}_{i,\text{ELB}}$, where the interest rate becomes zero. For example, when ϕ is large, the central bank reduces the nominal interest rate strongly to fight a small decline in inflation so that the ELB becomes binding even before the inflation rate turns negative. In this case, $\underline{r}_{\pi,\text{ELB}} < \underline{r}_{i,\text{REE}}$. When ϕ is close to but above one, the central bank tolerates large declines in the inflation rate so that inflation can turn negative even before the ELB is reached. In this case, $\underline{r}_{\pi,\text{ELB}} > \lim_{\phi\to+1} \underline{r}_{i,\text{REE}}(\phi)$, where we explicitly acknowledged that $\underline{r}_{i,\text{REE}}$ depends on ϕ.

Relatedly, even when the ELB on the nominal interest rate is slack, the inflation rate can be negative following a negative aggregate demand shock. The natural interest rate just needs to be below the cutoff

$$\underline{r}_{\pi,\text{no ELB}} \equiv \overline{r} + \overline{\pi}(1-\phi) - \overline{\pi}\left[\frac{1-\beta(1-\mu)}{\kappa}\sigma + 1\right]\mu.$$

We will use this cutoff in the next section.

3.2 Equilibrium with ZLB on Inflation Expectations

The preceding section makes clear that inflation and inflation expectations can be below zero in rational expectations equilibrium. This section adds the zero lower bound constraint on inflation forecasts and derives its consequences. We again use backward induction to solve for equilibrium. We continue assuming that the persistence of the negative interest rate shock is small enough so that the condition (12) holds, which will guarantee that the equilibrium is unique. The possibility of multiple equilibria is analyzed in Section 6.

First, consider the dates when the negative demand shock subsides. Formally, take all s such that $t+s \geq T$, where t is the date when the agent solves her problem, which can be either before T, at T, or after T. In this case, the agent forecasts $\widetilde{x}_{t+s} = 0$, $\widetilde{\pi}_{t+s} = \overline{\pi} > 0$, and $\widetilde{i}_{t+s} = \iota$ because these values satisfy equations (5)-(7).

Second, we solve for the forecasts at dates before the shock dissipates. Formally, take s such that $0 \leq t < t+s < T$. The Euler equation is

$$\widetilde{x}_L = -\frac{1}{\sigma\mu}\left[\max\left\{0, \overline{r} + \overline{\pi} + \phi\left(\widetilde{\pi}_L - \overline{\pi}\right)\right\} - (1-\mu)\widetilde{\pi}_L - \mu\overline{\pi} - \underline{r}\right], \tag{14}$$

where $\widetilde{x}_L$ and $\widetilde{\pi}_L$ are the forecasts of output gap and inflation in states with negative aggregate demand shock $r_t^n = \underline{r}$. The structure of uncertainty and the forecasts after the shock ends allow us to write the forecast from the Phillips curve (6) in a compact form

$$\widetilde{\pi}_L = \max\left\{\overline{\pi} + \frac{\kappa}{1-\beta(1-\mu)}\widetilde{x}_L, 0\right\}. \tag{15}$$

In the $(\widetilde{x}_L, \widetilde{\pi}_L)$ space, the modified Phillips curve features a horizontal part as can be seen in Figure 12.

Depending on the parameters of the model, the economy can be in one of the four states: (i) neither of the two constraints binds, (ii) only the ELB constraint binds, (iii) only the ZLB on inflation forecasts binds, (iv) both constraints bind. We consider these cases in turn.

No constraint binds. If the constraints do not bind, then the solution to the equilibrium equations is given by equations (9) and (10), which express output gap and inflation under rational expectations when the ELB on the nominal interest rate does not bind. Figure 12 presents the forecast Euler equation (curve EE') and the forecast Phillips curve (curve PC). Forecast inflation and output gap are in point A_1. Because the ZLB on inflation expectations does not bind, the realized output gap and inflation are identical to expected values under $r_t^n = \underline{r}$. For this equilibrium to occur, the natural interest rate must be larger than $\max\{\underline{r}_{i,\text{REE}}, \underline{r}_{\pi,\text{no ELB}}\}$. The condition that the natural interest rate is larger than $\underline{r}_{i,\text{REE}}$ ensures that the shock is small enough that it does not trigger the ELB on the nominal interest rate, while the requirement that the interest rate is higher than $\underline{r}_{\pi,\text{no ELB}}$ limits the size of the shock so that inflation forecast does not reach its zero lower bound.

Only the ELB binds. In the case with slack inflation expectations constraint, realized output gap and inflation again coincide with their forecasts. The solution is similar to the one presented in Section 3.1. Specifically, output gap and inflation are given by equations (9) and (11). Graphically, the expected Euler equation and the Phillips curve are shown in Figure 12 by curves EE'' and PC, respectively. They intersect in point A_2, where the ZLB on inflation expectations does not bind. Importantly, for this case to be possible in equilibrium, the natural interest rate shock must satisfy $\underline{r} \in (\underline{r}_{\pi,\text{ELB}}, \underline{r}_{i,\text{REE}}]$ and the parameters of the model must ensure $\underline{r}_{\pi,\text{ELB}} < \underline{r}_{i,\text{REE}}$, which states that the natural interest rate that makes inflation negative under the ELB is lower than then natural interest rate that makes the ELB constraint binding.

Only the ZLB on inflation expectations binds. We now turn to the case when the inflation forecast is zero, i.e., $\widetilde{\pi}_{t+s} = \widetilde{\pi}_L = 0$, while the lower bound on the interest rate is slack. The model parameters need to satisfy $\underline{r}_{i,\text{REE}} < \underline{r}_{\pi,\text{no ELB}}$, which is equivalent to $\bar{r} > \overline{\pi}(\phi - 1)$, and the size of the interest rate shock need to fall in the interval $(\underline{r}_{i,\text{REE}}, \underline{r}_{\pi,\text{no ELB}}]$ for this situation to occur in equilibrium. Figure 13 depicts such a scenario, where the kink in the Euler equation is below the zero inflation level and the leftward shift in the Euler equation is large enough to generate the ZLB on expected inflation.

The future Euler equation (5) and the Phillips curve (6) intersect at

$$\widetilde{x}_L = \frac{1}{\mu\sigma}\left[(\phi + \mu - 1)\,\overline{\pi} + \underline{r} - \overline{r}\right], \tag{16}$$

The forecast output gap in equation (16) is negative. Formally, $(\phi + \mu - 1)\,\overline{\pi} + \underline{r} - \overline{r} < 0$ when $\underline{r} < \underline{r}_{\pi,\text{no ELB}}$, which ensures that the ZLB on inflation forecast binds. However, note that the forecast output gap is higher than the one under rational expectations because the inflation rate expectations are constrained from below. This is clear from Figure 13 where the rational expectation equilibrium would feature a completely straight Phillips curve even when expected inflation is negative.

Conditional on zero inflation forecast and output gap forecast in equation (16), we compute the realized output gap and inflation by taking output gap and inflation forecasts as given and solving for the intersection of the realized Euler equation

$$x_L = -\frac{1}{\sigma}\left[\max\left\{\overline{r} + \overline{\pi} + \phi\left(\pi_L - \overline{\pi}\right), 0\right\} - \overline{\pi} - \underline{r}\right] + (1-\mu)\left(\widetilde{x}_L + \frac{\widetilde{\pi}_L - \overline{\pi}}{\sigma}\right), \tag{17}$$

and the realized Phillips curve

$$\pi_L = \overline{\pi} + \kappa x_L + \frac{\beta\theta(1-\mu)}{1-\beta\theta(1-\mu)}\left[\kappa\widetilde{x}_L + \frac{1-\theta}{\theta}\left(\widetilde{\pi}_L - \overline{\pi}\right)\right]. \tag{18}$$

The realized Euler equation (17) and the Phillips curve (18) are written conditional on any level of output gap and inflation forecasts and not only those found on the previous step of the backward induction procedure. This is to highlight how expectations influence current determination of the equilibrium variables. In particular, the second term in equation (17) and the third term in equation (18) formalize this forecasts' effect.

There are two possibilities in equilibrium: the ELB on the nominal interest rate binds or it is slack. First, if the natural interest rate falls below the cutoff

$$\begin{aligned}\underline{r}_{i,\text{ZLB}} \equiv &-\frac{\mu\sigma\left[1-\beta\theta(1-\mu)\right] - \phi\kappa(1-\mu)\left(1+\beta\theta\mu\right)}{\phi\kappa}\overline{r} \\ &-\frac{\mu\left\{\sigma\left[1-\beta\theta(1-\mu)\right] + \kappa\phi\right\} + (1-\mu)\phi\left[\kappa\left(\phi-1\right)\left(1+\beta\theta\mu\right) - \mu\sigma\beta\left(1-\theta\right)\right]}{\phi\kappa}\overline{\pi},\end{aligned}$$

then the ELB binds and equation (17) yields the actual output gap of

$$x_L = \frac{1-\mu}{\mu\sigma}\left[(\phi + \mu - 1)\,\overline{\pi} + \underline{r} - \overline{r}\right] + \frac{1}{\sigma}\left(\mu\overline{\pi} + \underline{r}\right).$$

and the equilibrium inflation rate is pinned down by the Phillips curve (18).[11]

This case is interesting. Despite the fact that agents do not expect to face the ELB on the interest rate under negative aggregate demand shock, the ELB actually binds. The reason is that when forecasting the future, the agents mistakenly believe that inflation will not fall below zero, which limits its impact on the forecast interest rate. In reality, however, the inflation rate does fall below zero pushing the interest to its lower bound.

Second, when the natural interest rate $\underline{r}$ is above $\underline{r}_{i,\text{ZLB}}$, the ELB does not bind in equilibrium and the realized output gap is

$$x_L = \frac{1}{\mu\left(\phi\kappa+\sigma\right)}\left\{\left[\left(\phi+\mu-1\right)\overline{\pi}+\underline{r}-\overline{r}\right]\left(1-\frac{\beta\theta(1-\mu)}{1-\beta\theta(1-\mu)}\cdot\frac{\phi\kappa}{\sigma}\right)-\mu\phi\frac{1-\beta(1-\mu)}{1-\beta\theta(1-\mu)}\overline{\pi}\right\},$$

and the inflation rate is again given by equation (18).

The requirement that the interest rate shock is below the cutoff $\underline{r}_{\pi,\text{no ELB}}$ ensures that the realized inflation rate π_L is negative.

Both constraints bind. Finally, when agents forecast that both constraints bind at the same time, the intersection of the expected Phillips curve (6) and expected Euler equation (5), depicted in Figure 12 by curves PC and EE''', is $\widetilde{\pi}_L = 0$ and $\widetilde{x}_L = \left(\mu\overline{\pi}+\underline{r}\right)/\left(\mu\sigma\right)$. Point A_3 on the figure represents this solution. The following condition on the parameters of the model has to be satisfied for the ELB to bind

$$\overline{r} \leq (\phi-1)\overline{\pi}.$$

Importantly, this condition does not involve the size of the shock $\underline{r}$, it just follows from plugging zero inflation rate in the Taylor rule (7). For inflation expectations to be constrained at zero, the shock $\underline{r}$ should be below the cutoff

$$\underline{r}_{\pi,\text{ELB}} \equiv -\mu\frac{\sigma\left[1-\beta(1-\mu)\right]+\kappa}{\kappa}\overline{\pi} < 0,$$

which follows from combining the solution to the forecast output gap and the Phillips curve (6).

To compute the realized inflation and output gap, we guess and then verify that the ELB constraint binds contemporaneously. The Euler equation (17) and the Phillips curve

[11] The fact that $\underline{r}_{i,\text{REE}} < \underline{r}_{\pi,\text{no ELB}}$ necessarily implies that $\underline{r}_{i,\text{ZLB}} < \underline{r}_{\pi,\text{no ELB}}$. Furthermore, $\underline{r}_{i,\text{ZLB}} > \underline{r}_{i,\text{REE}}$ when $\mu\sigma\beta < (\phi-1)\,\kappa$. Thus there are model parameters for which the set $(\underline{r}_{i,\text{REE}}, \underline{r}_{\pi,\text{no ELB}}]\bigcap(-\infty, \underline{r}_{i,\text{ZLB}}]$ is non-empty.

(18) yield

$$x_L = \widetilde{x}_L = \frac{1}{\sigma\mu}\left(\mu\overline{\pi} + \underline{r}\right), \tag{19}$$

$$\pi_L = \frac{\left[1 - \beta(1-\mu) + \frac{\kappa}{\sigma}\right]\overline{\pi} + \frac{\kappa}{\sigma\mu}\underline{r}}{1 - \beta\theta(1-\mu)} < 0. \tag{20}$$

Note that the reason why output gap equals expected output gap under $r_t^n = \underline{r}$ is because the nominal interest rate and expected inflation are zero. As a result, the realized Euler equation is identical to expected Euler equation.

Given this solution, the interest rate prescribed by the Taylor rule is

$$i_L = \iota + \phi(\pi_L - \overline{\pi}) \leq 0,$$

which verifies our assumption that the ELB binds even contemporaneously. Furthermore, the condition that the interest rate shock is below the cutoff $\underline{r}_{\pi,\text{ELB}}$ guarantees that the realized inflation is below zero.

An important property of the solution in equations (19) and (20) is that it does not explode when probability μ approaches its cutoff where the denominator $\vartheta(\mu)$ in equation (11) becomes zero. This property contrasts with the FIRE case. Intuitively, a typical New-Keynesian deflationary spiral under the ELB on the nominal interest rate is turned off when inflation forecast remains at zero. However, note that as μ declines, output gap and inflation still become large in absolute terms but negative because $\underline{r} < \underline{r}_{\pi,\text{ELB}} < 0$. This is because the effect of the low future natural interest rate still propagates and accumulates through the Euler equation. It is the amplification mechanism of the Phillips curve that is turned off.

Finally, we emphasize that while deflation occurs at the ELB on nominal interest rate, it is milder than that without the ZLB constraint on inflation expectations.

4 Unconventional Monetary Policies

The experience with unconventional monetary policies during the Great Recession and its aftermath left many economists puzzled by apparent disconnect between the powers of these policies in many macro models and the lack of such powers in reality (e.g., the so-called "forward guidance puzzle", see Del Negro, Giannoni and Patterson, 2012). In this section, we show that unconventional policies that have large effects during the ELB under rational expectations have weaker effects when the ZLB on inflation expectations binds.

4.1 Forward Guidance

We start with the policy of forward guidance, which is an attempt to affect current aggregate economic activity by promising certain policy actions in the future, most notably, to keep a low interest rate for long. We show that forward guidance is less powerful when the zero-lower bound on inflation expectation binds. Intuitively, because inflation expectations do not respond to forward guidance, the effect is confined to the response of output gap to the future promised interest rate changes through the Euler equation.

To formally illustrate this result, assume that after the ELB ends, the central bank keeps the interest rate at zero for some time. The probability of continuation of zero interest rate policy is time dependent. Specifically, with probability $1 - p_0$, the interest stays at zero, while with probability p_0, it starts following the Taylor rule in period $t = T$, when the shock to the natural interest rate ends. In all later periods $t > T$, these probabilities change to $1 - p_1$ and p_1, respectively. It will be useful to distinguish between p_0 and p_1 in the analysis below. In particular, probability p_1 will not only affect the duration of the "low for long" policy but also its impact on output gap and inflation after the negative interest rate shock dissipates. At the same time, probability p_0 only affects the initial chance of low interest rate policy continuation without affecting its impact on output gap and inflation in periods $t \geq T$. As a result, we will be able to study the output effect of an increase in the chance of the low interest rate policy conditional on its strength in $t \geq T$. Finally, we denote the (stochastic) period when the interest rate returns to the Taylor rule as T_{TR} and solve the model again by backward induction.

First, for any $t \geq T_{TR}$, we have $x_t = 0, \pi_t = \overline{\pi}, i_t = \iota$. Second, for $t \in [T, T_{TR})$, the Euler equation and the Phillips curve determine contemporaneous and future forecast values of inflation and output gap

$$\begin{aligned} x_{FG} &= \frac{[1 - \beta(1 - p_1)]\,(\overline{\pi} + \overline{r})}{p_1 \sigma\,[1 - \beta(1 - p_1)] - (1 - p_1)\kappa}, \\ \pi_{FG} &= \overline{\pi} + \kappa \frac{x_{FG}}{1 - \beta(1 - p_1)} > \overline{\pi}, \end{aligned}$$

where x_{FG} and π_{FG} are output gap and inflation during the period of stimulative monetary policy. A unique bounded equilibrium exists only when $(1 - p_1)\kappa < p_1 \sigma\,[1 - \beta(1 - p_1)]$, that is, the central bank does not keep the interest rate low for too long after the exit of the ELB. Note that because inflation π_{FG} is positive, the actual inflation coincides with inflation forecast because the ZLB on inflation expectations does not bind.

Third, consider periods $t \in [0, T)$. To jump to the core of our analysis, we focus on the case when both the ELB on the nominal interest rate and the ZLB on inflation expectations bind provided the natural interest rate is low and absent forward guidance policy.

Furthermore, we assume that forward guidance is sufficiently modest so that the two constraints continue to bind even when the policy is implemented. As a result, the forecasts of inflation is $\widetilde{\pi}_L = 0$ and of output gap is

$$\widetilde{x}_L = \frac{1}{\sigma\mu}\left(\mu\overline{\pi} + \underline{r}\right) + (1 - p_0)\left[x_{FG} + \frac{\pi_{FG} - \overline{\pi}}{\sigma}\right],$$

The bottom panel of Figure 14 plots the forecast Euler equation and the Phillips curve that generate the above values of inflation expectation and output gap in the state with $r_t^n = \underline{r}$. It is instructive to compare the effect of the forward guidance on expectations when the ZLB on inflation expectations binds with the case of rational expectations, which we plot in the top panel of Figure 14. It is evident that for the same size of forward guidance policy, expected output gap increases by less under the ZLB on inflation expectations. There are two reasons for this, both of which work through the Phillips curve. First, when inflation expectations are stuck at zero, forward guidance does not directly affect the flat part of the Phillips curve and inflation forecast for that state remains at zero. Second, forward guidance does not influence expected inflation indirectly through an increase in output gap because the expected inflation is stuck at zero.

The realized output gap is

$$x_L = \widetilde{x}_L = x^{\text{no FG}} + (1 - p_0)\left[x_{FG} + \frac{\pi_{FG} - \overline{\pi}}{\sigma}\right], \tag{21}$$

where the first term of the equation $x^{\text{no FG}} \equiv \left(\mu\overline{\pi} + \underline{r}\right)/(\sigma\mu)$ is the output gap without forward guidance. An example of the evolution of economic variables is presented in Figure 15 where we set random realizations of T to 5 and T_{TR} to 8. The last term in equation (21) quantifies the effect of forward guidance. It is proportional to the probability $1 - p_0$ that the policy will be implemented as well as its strength summarized by the term in the square brackets.

The next step is to compare expression (21) to its rational expectations counterpart. Assuming that the ELB constraint binds, we solve for output gap

$$x_L = x_L^{\text{no FG, REE}} + \frac{\sigma\mu\left[1 - \beta(1-\mu)\right](1 - p_0)}{\sigma\mu\left[1 - \beta(1-\mu)\right] - (1-\mu)\kappa}\left[x_{FG} + \frac{1}{1 - \beta(1-\mu)} \cdot \frac{\pi_{FG} - \overline{\pi}}{\sigma}\right], \tag{22}$$

where $x_L^{\text{no FG, REE}}$ is the rational expectations output gap in the ELB expressed in equation (11). It is clear that the strength of the forward guidance effect on output gap in rational expectations equilibrium, the second term in equation (22), is larger than that in equilibrium where inflation expectations are constrained to be non-negative, the second term in equation (21). Formally, this is because the coefficient multiplying the square bracket in equation (22) is larger than one, and the coefficient multiplying the deviation of inflation from its target is also above one. Intuitively, inflation forecasts in the ELB state are not affected by forward guidance announcements (unless they are sufficiently stimulative to lift inflation expectations above zero) and the feedback from higher expected inflation on the ex ante real interest rate and, hence, real economic activity is switched off.

Interestingly, the strength of forward guidance increases with the persistence of the ELB episode in rational expectations equilibrium because $\partial(x_L - x_L^{\text{no FG, REE}})/\partial(1-\mu) > 0$ in equation (22), while it does not depend on the ELB duration when inflation forecasts are constrained at zero because $\partial(x_L - x_L^{\text{no FG}})/\partial(1-\mu) = 0$ in equation (21). The strength depends on the duration of the ELB in the rational expectation equilibrium because an increase in output gap boosts inflation, which, in turn, increases output gap. This two-way feedback is stronger the longer agents expect the economy to remain at the ZLB. With the constraint on expected inflation, forward guidance increases expected inflation outside of the ELB, but then output gap does not affect inflation expectation during the ELB.

4.2 Negative interest rates

This section investigates the possibility of allowing the short term nominal interest rate to go below its lower bound. When the interest rate is allowed to be negative, the solution to the output gap is similar to the one in equation (19) with the only addition being $\epsilon > 0$ inside the brackets, which represents the value by which the interest rate can go negative. Taking the difference between the output gap with the interest rate set at a small negative value $-\epsilon$ and zero and assuming that the ZLB on inflation expectation binds, we obtain

$$\Delta \equiv x_L(i_L = -\epsilon) - x_L(i_L = 0) = \frac{1}{\mu\sigma}\epsilon.$$

The difference under rational expectations is

$$\Delta^{REE} \equiv x_L(i_L = -\epsilon) - x_L(i_L = 0) = \frac{1-\beta(1-\mu)}{[1-\beta(1-\mu)]\mu\sigma - (1-\mu)\kappa}\epsilon.$$

It is clear that $\Delta^{REE} > \Delta$. Interestingly, in both cases the effectiveness of this policy (slightly negative interest rate) depends on the duration of the ELB. The notable difference, however, is that the ELB needs to be permanent for the negative interest rate to become infinitely effective under the ZLB on inflation expectations, while under rational expectations it occurs for a smaller value of the ELB persistence $1-\mu$ that satisfies $\theta = 0$.

4.3 Optimal Inflation Target

Because the ELB on expected inflation makes unconventional policy tools potentially less powerful and hence the cost of deflation spirals greater, one may think that the ELB on expected inflation justifies raising the inflation target so that the central bank is less likely to run into this constraint. To keep our analysis close to the standard three-equation New Keynesian model, we assumed that firms can index their prices with the trend level of inflation. While this assumption makes the model tractable, it greatly attenuates price dispersion, a key cost of positive trend inflation (Coibion *et al.*, 2012), thus strengthening the rationale for raising the target. On the other hand, the risk of deflation is greatly reduced when economic agents do not believe that deflation is possible. Indeed, our analysis suggests that in this case the possibility of sunspot variation is attenuated greatly. Because the cost of deflation is highly nonlinear in the duration of nominal interest rates at the ELB, the benefit of raising the target is reduced as well. As a result, the calculus of tradeoffs involved in establishing the optimal inflation target is more nuanced and depends on parameter values (frequency of price changes, frequency and size of shocks that put the economy at the ELB for nominal interest rates, etc.) as well as other details of the model.

5 Aggregate Supply and Demand Shocks

The effective lower bound on the nominal interest rate changes the response of the economy to shocks in standard New Keynesian models. This sections investigates how the presence of the zero lower bound on inflation expectations alters this prediction by considering shocks to the Phillips curve and the Euler equation.

5.1 Aggregate Supply Shocks

Inflationary Phillips-curve shocks, which we will refer to as negative aggregate supply shocks, are contractionary when the Taylor rule is active but can be expansionary under the ELB when agents have FIRE (Eggertsson, 2011, 2012). For example, shocks that increase firms' marginal costs for some time, such as higher taxes or positive shocks to intermediate input prices, tend to increase inflation and expectations thereof. With an active Taylor rule where the nominal interest rate responds more than one-to-one to changes in inflation rate, the real interest rate increases thus depressing current consumption and output gap. By contrast, when the ELB on the nominal interest rate binds, the nominal interest rate does not change, forcing the real interest rate to go down due to an increase in expected inflation. This, in turn, has a positive effect on consumption demand and output gap. We show that this logic does not hold when the zero lower bound on inflation expectations binds.

To formally illustrate the effects of negative aggregate supply shocks, we add shock u_t to the contemporaneous and expected Phillips curves (3) and (6) as follows

$$\pi_t = \overline{\pi} + \kappa x_t + \sum_{k=1}^{\infty} (\beta\theta)^k \mathbb{E}_t \left[\kappa \widetilde{x}_{t+k} + u_{t+s} + \frac{1-\theta}{\theta} \left(\widetilde{\pi}_{t+k} - \overline{\pi} \right) \right],$$

$$\widetilde{\pi}_{t+s} = \max \left\{ \overline{\pi} + \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_{t+s} \left[\kappa \widetilde{x}_{t+s+k} + u_{t+s+k} \right] + \frac{1-\theta}{\theta} \sum_{k=1}^{\infty} (\beta\theta)^k \mathbb{E}_{t+s} \left[\widetilde{\pi}_{t+s+k} - \overline{\pi} \right], 0 \right\},$$

where $s \geq 1$.

The Euler equation does not directly depend on the shock to the Phillips curve. To generate the ELB on the nominal interest rate and the ZLB on inflation expectations, we continue using a sufficiently large negative shock to the natural interest rate $r_t^n = \underline{r}$ that reverts back to its steady state value $\overline{r}$ with probability μ. In addition, we assume that shock u_t is zero in steady state and positive, i.e., $u_t = u > 0$, as along as the natural interest rate equals its low value of $\underline{r}$.[12]

The top panel in Figure 16 plots the determination of inflation and output gap under FIRE. The shock pushes the Phillips curve up from the solid to the dashed line. The equilibrium changes from point A_1 to point A_2, which features a higher output gap and inflation. The bottom panel of Figure 16 describes the determination of expected output gap and inflation when the ZLB on inflation expectations binds. This diagram illustrates that when shock u_t is small enough, it fails to move the intersection (point A_1 on the figure) of the Euler equation and the Phillips curve. As a result, the households continue to expect zero inflation and the same value of output gap, which is formally expressed in equation (19). Furthermore, the actual realized output gap will not change as well because the nominal interest rate is at its ELB, and expected output gap and inflation rates do not change. Meanwhile, the realized inflation does go up. Formally, the contemporaneous Phillips curve together with the solved values for forecast inflation and output gap give

$$\pi_L = \frac{\left[1 - \beta(1-\mu) + \frac{\kappa}{\sigma}\right] \overline{\pi} + \frac{\kappa}{\sigma\mu} \underline{r} + u}{1 - \beta\theta(1-\mu)}.$$

The last expression shows that the actual inflation rate goes up following the shock.

The above results have some empirical support. Wieland (2019) finds that the respon-

[12]It is straightforward to extend the analysis to the case when shocks u_t is more or less persistent than the shocks to the natural interest rate.

dents of the Michigan Survey of Consumers do not change their inflation expectations in response to negative aggregate supply shocks (proxied by negative *oil* supply shocks) at the ELB on the nominal interest rate. At the same time, professional forecasters (represented by the Consensus Economics average inflation forecast) and financial markets (represented by the difference between the nominal and real rates) do increase their inflation expectations following negative aggregate supply shocks (proxied either by the Great East Japan earthquake or by negative oil supply shocks) in Japan. This is consistent with our observation that the realized inflation goes up and that professional forecasters have much more accurate inflation expectations (Section 2.6). At the same time, Wieland (2019) finds that aggregate output drops after negative supply shocks. This can be consistent with the prediction of our model that output gap is unchanged and the fact that negative supply shocks reduce natural output.

5.2 Aggregate Demand Shocks

In standard New Keynesian models, aggregate demand shocks, such as an increase in government purchases, can have a stronger impact on the economy under the ELB on the nominal interest rate than in the normal times when the ELB is slack. Moreover, this difference between the ELB and normal times effects can be extremely large (Woodford, 2010; Christiano, Eichenbaum and Rebelo, 2011). The reason for the difference in the effects is that the nominal interest rate does not react to changes in inflation following shocks when the ELB on nominal interest rate binds. In addition, persistent shocks affect expected inflation, which, in turn, changes current output gap and inflation, magnifying the direct effect of the shock.

This section focuses on one example of aggregate demand shocks—changes in government purchases. We show that the ZLB on inflation expectations keeps the result that the output multiplier is larger under the ELB on the nominal interest rate than under active Taylor rule. At the same time, the amplification of persistent aggregate demand shocks is attenuated when the inflation expectations are stuck at zero.

With an additional source of spending in the economy, we need to modify the Euler equation and the Phillips curve. The contemporaneous and future Euler equations become

$$x_t - g_t = \mathbb{E}_t[\widetilde{x}_{t+1} - g_{t+1}] - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t[\widetilde{\pi}_{t+1}] - r^n_t\right),$$
$$\widetilde{x}_{t+s} - g_{t+s} = \mathbb{E}_{t+s}\left[\widetilde{x}_{t+s+1} - g_{t+s+1}\right] - \frac{1}{\sigma}(\widetilde{i}_{t+s} - \mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1}\right] - r^n_{t+s}),$$

where $s \geq 1$, $g_t \equiv (G_t - \overline{G})/\overline{Y}$, G_t is government purchases, and $\overline{G}$ is its steady-state value. The current and expected Phillips curves are

$$\pi_t = \overline{\pi} + \kappa(x_t - \Gamma g_t) + \sum_{k=1}^{\infty}(\beta\theta)^k \mathbb{E}_t\left[\kappa(\widetilde{x}_{t+k} - \Gamma g_{t+k}) + \frac{1-\theta}{\theta}(\widetilde{\pi}_{t+k} - \overline{\pi})\right],$$

$$\widetilde{\pi}_{t+s} = \max\left\{\overline{\pi} + \kappa\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_{t+s}\left[\widetilde{x}_{t+s+k} - \Gamma g_{t+s+k}\right] + \frac{1-\theta}{\theta}\sum_{k=1}^{\infty}(\beta\theta)^k \mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+k} - \overline{\pi}\right], 0\right\},$$

where $s \geq 1$, $\Gamma < 1$ is the government spending output multiplier under flexible prices.[13] Observe that a change in government consumption not only directly affects the Euler equations but also the Phillips curves. This is because higher government spending makes households poorer due to an increased tax burden and, hence, more willing to work (due to a negative income effect on leisure). This reduces firms' marginal costs and pushes inflation down. We assume that $g_t = g > 0$ as long as the natural real interest rate is $\underline{r}$, and $g_t = 0$ when the interest rate reverts back to its steady-state value, and that all agents are aware of this law of motion.

Figure 17 graphically presents the solution. The upper panel shows the case of FIRE. The Euler equation and the Phillips curve both move to the right. Importantly, because the Euler equation shifts to the right by g, while the Phillips curve shift to the right by $\Gamma g < g$, the intersection of the shifted curves is in point A_2 where both inflation and output gap are larger than those in point A_1. Furthermore, because a change in inflation is positive, an increase in output gap is larger than g, implying that the fiscal multiplier is larger than one. Formally, the output gap in point A_2 of the upper panel of Figure 17 is

$$x_L = \frac{\left[1-\beta(1-\mu)\right]\mu\sigma - (1-\mu)\kappa\Gamma}{\left[1-\beta(1-\mu)\right]\mu\sigma - (1-\mu)\kappa} g + x_L^{\text{no } g, REE},$$

where $x_L^{\text{no } g, REE}$ is the rational expectations output gap under the low value of natural interest rate and no change in government consumption, expressed in equation (11). It is evident that the coefficient that multiplies the government spending in the first term of the last formula is greater than one as long as the condition for the existence of the unique equilibrium (12) is satisfied. What is more, when μ increases so that the denominator $\left[1-\beta(1-\mu)\right]\mu\sigma - (1-\mu)\kappa$ becomes arbitrarily close to zero, the multiplier $\Delta x_L / \Delta g$ becomes unbounded.

We now contrast the FIRE case to the case of the ZLB on inflation expectations. The bottom panel of Figure 17 plots the expected Euler equation and the Phillips curve before

[13] Γ can be obtained by differentiating the intra-temporal optimality condition of the representative household under flexible prices. When the household preferences take the form $u(C_t) - v(N_t)$ and the production function is linear $Y_t = A_t N_t$, the intertemporal optimality condition $u'(Y_t - G_t) = v'(Y_t/A_t)/A_t$ yields $\Gamma \equiv dY_t/dG_t = -u''(Y_t - G_t)/[-u''(Y_t - G_t) + v''(Y_t/A_t)/A_t^2]$.

the fiscal shock (solid curves) and with positive government consumption (dashed lines). The expected output gap and inflation in the low natural interest rate state go from point A_1 to point A_2, implying that the inflation expectations does not change, while output gap increases by g. Expected inflation does not move because the shock is small enough so that the economy is still at the ZLB on inflation expectations. Expected output gap moves by exactly g because the interest rate and expected inflation are constant at zero. This is in contrast to the case of FIRE beliefs where expected (and realized) output gap increases by more than g.

Realized inflation and output gap can be obtained by jointly solving contemporaneous Phillips curve and the Euler equations conditional on the above expectations. We find

$$x_L = \widetilde{x}_L = \widetilde{x}_L^{\text{no } g} + g,$$
$$\pi_L = \pi_L^{\text{no } g} + \frac{\kappa(1-\Gamma)}{1-\beta\theta(1-\mu)} g,$$

where $\widetilde{x}_L^{\text{no } g}$ and $\pi_L^{\text{no } g}$ (expressed in equations (19) and (20)) are the values of output gap and inflation under the low natural interest rate, ELB on nominal interest rate, and ZLB on expected inflation. There are two main messages from inspecting the two formulas. First, the output multiplier is one and the inflation response is positive. Second, the output multiplier value of one does not depend on the duration of the ELB on the nominal interest rate. At the same time, inflation response increases with more persistent ELB episode, i.e., with larger $1-\mu$. However, this sensitivity does not blow up as the ELB episode becomes more and more persistent.

The predictions of our model are consistent with empirical findings in Miyamoto, Nguyen and Sergeyev (2018) for Japan. First, they show that the fiscal multiplier is significantly larger during the ELB on the nominal interest rate than in normal time in Japan. However, the authors cannot reject that the ELB output multiplier is equal to one. Second, they find that inflation responds significantly more during the ELB episode than in normal times following a shocks to government consumption. In addition, inflation forecast by the Japan Center of Economic Research also increases by more during the ELB episode than during the normal period. This is again consistent with the observation that professional forecasters tend to have more accurate inflation forecasts.

6 Self-fulfilling Liquidity Traps

Sections 3-5 analyzed theoretical implications of the ZLB on inflation expectations in a model where a low inflation state, which generated a liquidity trap, resulted from a fundamental shock to a natural rate of interest. However, this is not the only possibility. In a seminal contribution, Benhabib, Schmitt-Grohe and Uribe (2001a); Benhabib, Schmitt-Grohé and Uribe (2001b) showed that self-fulfilling liquidity traps with deflation could occur without any fundamental shocks when the monetary policy rate reacts strongly to inflation unless it reaches the ELB.[14] This section shows that the ZLB on inflation expectation qualitatively changes predictions of a simple New Keynesian model with confidence shocks.

A confidence-driven liquidity trap steady state with deflation is no longer possible under the ZLB on inflation expectations. With FIRE, the 3-equation New Keynesian model from Section 3 has a standard steady state where inflation equals its target value of $\overline{\pi}$, the output gap is zero, and the nominal interest rate is ι. However, there is an additional steady state that satisfies the three equations. In this steady state, the nominal interest rate is at the ELB, i.e., $i_t = 0$. Because of the Fisher identity, expected and current inflation rates are *negative* and equal to $-\bar{r}$. Finally, the Phillips curve requires that output gap is $-(1-\beta)(\bar{r}+\overline{\pi})/\kappa$. An important assumption leading to a self-fulfilling liquidity trap steady state with deflation is that agents anticipate that inflation will be negative in the future. If, however, inflation expectations cannot fall below zero, this liquidity trap is no longer possible.

To further illustrate how the ZLB on inflation expectations changes the implications of the model with confidence shocks, we assume that the economy faces non-fundamental ("sunspot") shocks ξ_t that coordinate agents' beliefs. As in Mertens and Ravn (2014), sunspots can take two values: "pessimistic" ξ_L and "optimistic" ξ_H, with transition probabilities $\mathbb{P}(\xi_{t+1} = \xi_H | \xi_t = \xi_H) = 1$ and $\mathbb{P}(\xi_{t+1} = \xi_L | \xi_t = \xi_L) = 1 - \mu_\xi \in [0, 1]$. Hence the optimistic state is absorbing. It corresponds to the standard steady state with $(\pi_t, x_t, i_t) = (\overline{\pi}, 0, \iota)$. The economy starts with the "pessimistic" state ξ_L. Moreover, we allow for a fundamental shock to the natural real interest rate r_t^n. It takes the value of $\widehat{r}$ as long as the pessimistic sunspot lasts, after which r_t^n returns to its steady state value of $\bar{r}$.[15]

FIRE benchmark. We consider the FIRE assumption first. The equilibrium is described by the Euler equation (8), where we replace μ with μ_ξ and $\underline{r}$ with $\widehat{r}$, and by the Phillips curve (9), where μ is replaced by μ_ξ. The top panel of Figure 18 plots three positions of the Euler equation corresponding to the three values of the natural interest rate r'', r', and $\bar{r}$ such that $r'' < r' < \bar{r}$, and one Phillips curve in the (x_L, π_L)-space. We focus on the parameter values that make upward-sloping parts of the Euler equations flatter than the

[14] Mertens and Ravn (2014) analyze fiscal policy effects, and Bilbiie (2018) studies the positive and normative implications of monetary and fiscal policies in confidence-driven liquidity traps.

[15] It is not essential that the fundamental and sunspot shocks last the same number of periods. This assumption is straightforward to relax.

Phillips curve in Figure 18. This assumption creates a possibility of multiple equilibria. Specifically, we require

$$\vartheta(\mu_\xi) = \mu_\xi \sigma[1 - \beta(1 - \mu_\xi)] - (1 - \mu_\xi)\kappa < 0, \tag{23}$$

which is the opposite of the condition (12) that ensured a unique bounded equilibrium under fundamental shock to the natural interest rate. Condition (23) holds, for example, when the persistence $1 - \mu_\xi$ of the pessimistic state is high enough, which includes a permanent pessimistic state.

Absent natural interest rate shocks, the Euler equation is depicted with the solid line EE in the top panel of Figure 18, and it has two intersections with the Phillips curve. The intersection A_H is the standard steady state, while the intersection A_L features the ELB on the nominal interest rate. The fact that output gap and inflation are negative in point A_L explains why we call this state "pessimistic." A change in the natural interest rate to r' shifts the Euler equation leftward to the position EE'. This shift preserves the existence of two equilibria, which are in points A'_H and A'_L. If, instead, the Euler equation is in its position EE'' due to a sufficiently negative shock to the natural rate of interest, there is no equilibrium as the Phillips curve does not intersect with the Euler equation.[16]

ZLB on inflation expectations. When agents expect only non-negative inflation, the properties of self-fulfilling equilibria change in important ways. To demonstrate this, we solve for beliefs by looking for the intersection of the forecast Euler equation (14) and the forecast Phillips curve (15), where we again replace μ with μ_ξ and $\underline{r}$ with $\widehat{r}$. The bottom panel of Figure 18 depicts the three positions of the Euler equation and the Phillips curve that features the ZLB on inflation expectations.

Without fundamental shocks, the ZLB on inflation expectations removes the possibility of confidence-driven liquidity traps. To see this, consider the Euler equation EE in the bottom panel of Figure 18. An important feature of the Euler equation is that it intersects zero inflation at a positive output gap (point N in the figure). Even when the persistence of the pessimistic state is close to one, making the upward-sloping part of the Euler equation nearly flat, the intersection with zero inflation still occurs at a positive level of output gap.[17] Thus, there is only one intersection of the Euler equation and the Phillips curve in the bottom panel of Figure 18. This intersection is the standard steady state $(\widetilde{\pi}_L, \widetilde{x}_L) = (\overline{\pi}, 0)$. Given these beliefs, the actual realized inflation and output gap

[16] Ascari and Mavroeidis (2020) discuss in detail that both parameters and shock support restrictions are needed to ensure the existence of equilibria in New-Keynesian models.

[17] Formally, in the case when $\widetilde{i}_L = 0$ and $\widetilde{\pi}_L = 0$, the Euler equation is $\widetilde{x}_L = (1 - \mu_\xi)\widetilde{x}_L + \sigma^{-1}(\mu_\xi \overline{\pi} + \overline{r})$, implying that $\widetilde{x}_L$ is positive regardless the value of μ_ξ.

are $(\overline{\pi}, 0)$. Formally, these values solve equations (17) and (18), where $\mu = \mu_{\xi}$ and $\underline{r} = \bar{r}$. In light of this result, one can think that the ZLB on inflation expectations prevents self-fulfilling liquidity traps. This is, however, not the case if there are fundamental shocks in addition to sunspots, which we turn to next.

When confidence shocks occur together with fundamental shocks, the number of equilibria can increase. Specifically, if a negative shock to the natural interest rate shifts the forecast Euler equation to the position EE' in the bottom panel of Figure 18, there are three intersections with the forecast Phillips curve instead of just two as in the top panel of the same figure. The extra equilibrium is due to the flat part of the forecast Phillips curve, representing the ZLB on inflation expectations. Interestingly, both intersections A'_{LL} and A'_{L} feature the ELB on the nominal interest rate. This example illustrates that negative "aggregate demand" shocks can expose the economy to sunspot shocks instability.

Finally, when the natural interest rate shock is large enough so that the forecast Euler equation shifts to its position EE'', there is a unique equilibrium represented by point A''_{L}. This is in contrast to the case with the FIRE assumption where the equilibria do not exist under such a fundamental shock.

7 Conclusion

Is deflation on the horizon? This is a question that many central bankers are deeply concerned about in current economic conditions with depressed economy and persistently low inflation. And they should be as the Great Depression in the 1930s and Japan since the early 1990s are grim reminders of what deflation can inflict. Furthermore, once deflation takes hold over the minds, the escape may be difficult. For example, while discussing the Japanese experience at the 2018 ECB conference, Haruhiko Kuroda, the governor of the Bank of Japan, said, "The deflationary mindset that has become entrenched amongst people is quite tenacious and it will take time to completely dispel this mindset."

Fortunately, our findings offer some hope, at least in the short run. In contrast to professional forecasters, households do not anticipate deflation in the near future even when there is a deflation and/or professional forecasters project deflation. Specifically, in the cross-sectional distribution, few households predict deflation. The share of households predicting deflation shows little variation over time. Once pushed close to zero, households' inflation expectations are insensitive to variation in actual inflation. In short, there seems to be a zero lower bound (ZLB) on households' inflation expectations. Although the lack of high-quality surveys of firms' inflation expectations prevents us from having definitive conclusions, the available evidence suggests that the same ZLB on inflation expectations applies to firms as well.

Using the standard three-equation New Keynesian model as a starting point, we show that households' inflation expectations with a floor at zero can affect macroeconomic dynamics and the effectiveness of unconventional policy tools that rely on managing expectations. For example, if the policy rate is stuck at the effective lower bound (ELB) and inflation expectations are stuck at the ZLB, forward guidance will have weaker effects on the economy. Intuitively, with full-information rational expectations (FIRE), forward guidance typically raises inflation expectations even if the nominal interest rate is at the ELB. However, this can mean increasing inflation expectations from, say, -3 percent to -2 percent. If inflation expectations are stuck at the ZLB, then such an increase in FIRE expectations is infra-marginal. On the bright side, if inflation expectations can't fall below zero, the possibility of deflationary spirals can be attenuated and hence the policymaker may be in a situation that is less desperate than predicted by FIRE-based models.

While our analysis remains agnostic about the sources of the ZLB on inflation expectations, future work will likely have to rely on randomized controlled trials that create exogenous variation pushing households' inflation expectations toward zero. With such variation as well as additional survey data, one may hope to establish why households are reluctant to predict deflation. We also hope that future work will move beyond the Survey of Firms' Inflation Expectations and TANKAN and make progress in understanding whether and how ZLB on inflation expectations applies to firms. Finally, while the basic New Keynesian model offers remarkable tractability and thus important theoretical insights, future work can provide deeper quantitative assessments of ZLB on inflation expectations. But in any case, central bankers likely do not have to worry about dispelling households' deflationary mindset as households do not predict deflation on the horizon.

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Main Text Figures and Tables

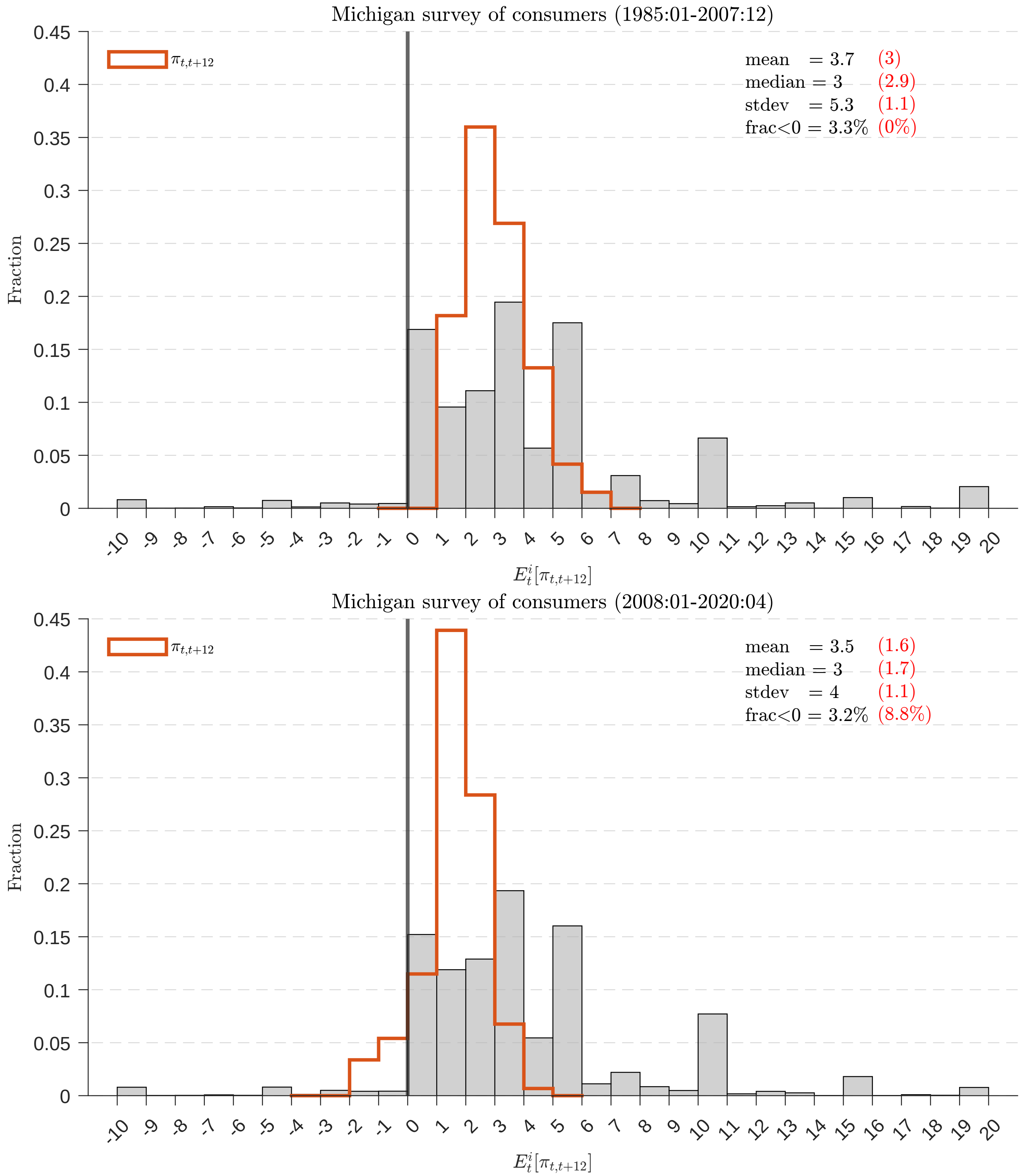

Figure 1: The top and bottom panels are the histogram of realized CPI inflation (in red) and the Michigan Survey of Consumers inflation expectations (in gray) between 1985:01 and 2007:12, and between 2008:01 and 2020:04, respectively.

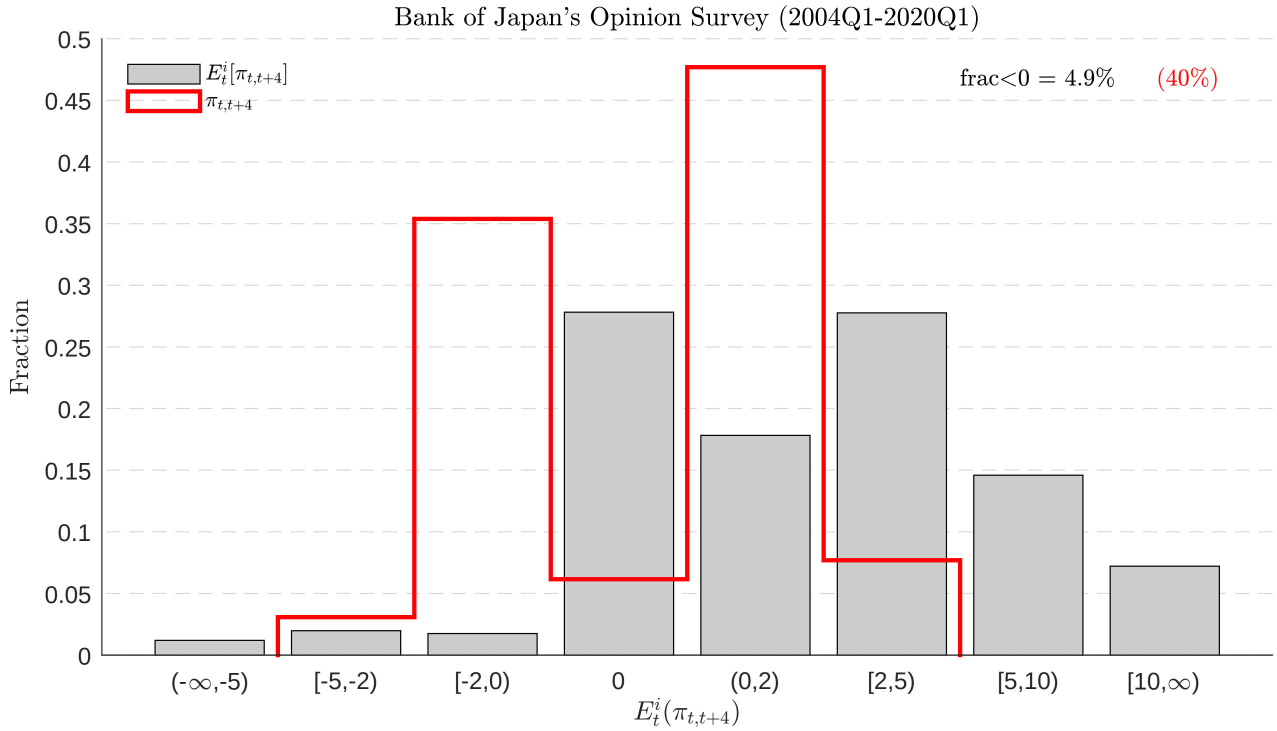

Figure 2: The histogram of realized CPI inflation (in red) and Bank of Japan's Opinion Survey one year ahead inflation expectations (in gray) between 2004Q1 and 2020Q1. The values of realized inflation in the interval [-0.2%;0.2%] are set to zero.

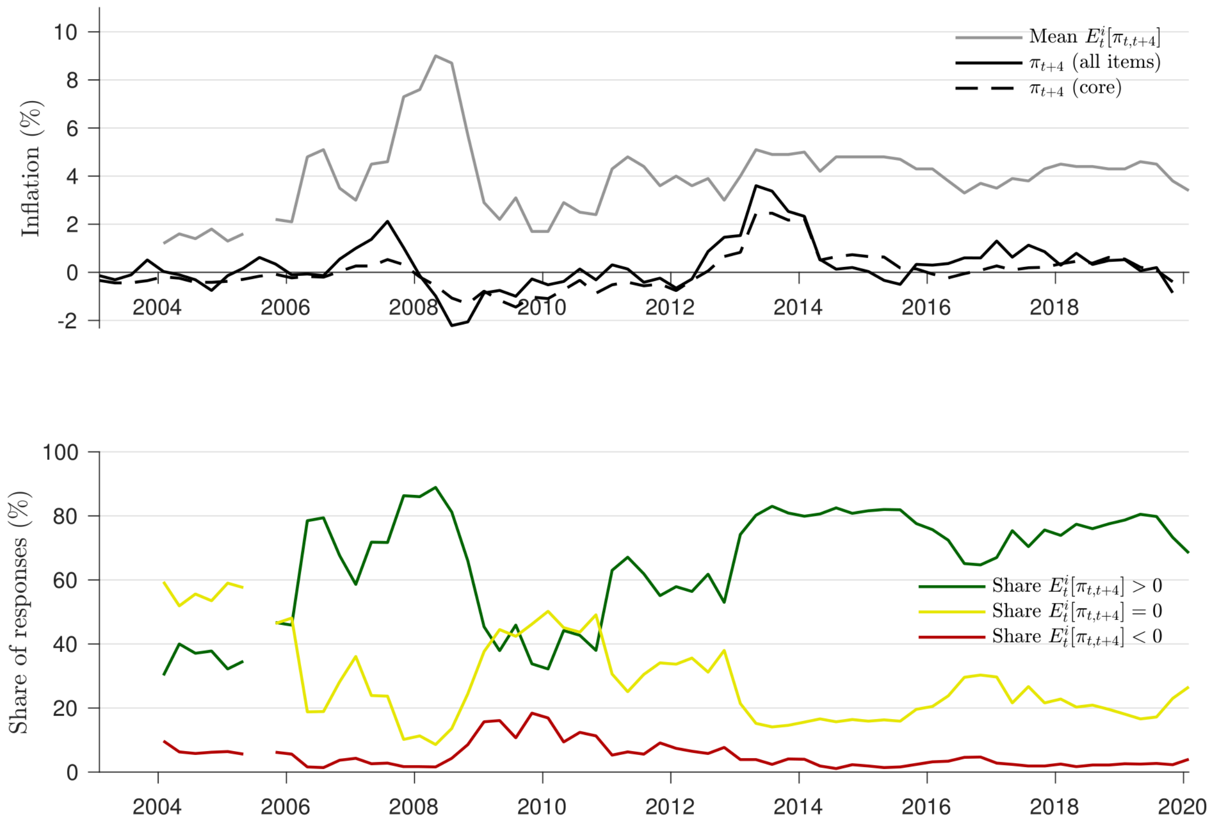

Figure 3: The top panel of the figure shows Japanese realized inflation (bold black line), Japanese realized core inflation (dashed black line), and a median inflation forecast from Bank of Japan's Opinion Survey (solid gray line). The bottom panel presents shares of Bank of Japan's Opinion Survey participants who expect positive inflation (green line), zero inflation (yellow line), and negative inflation (red line).

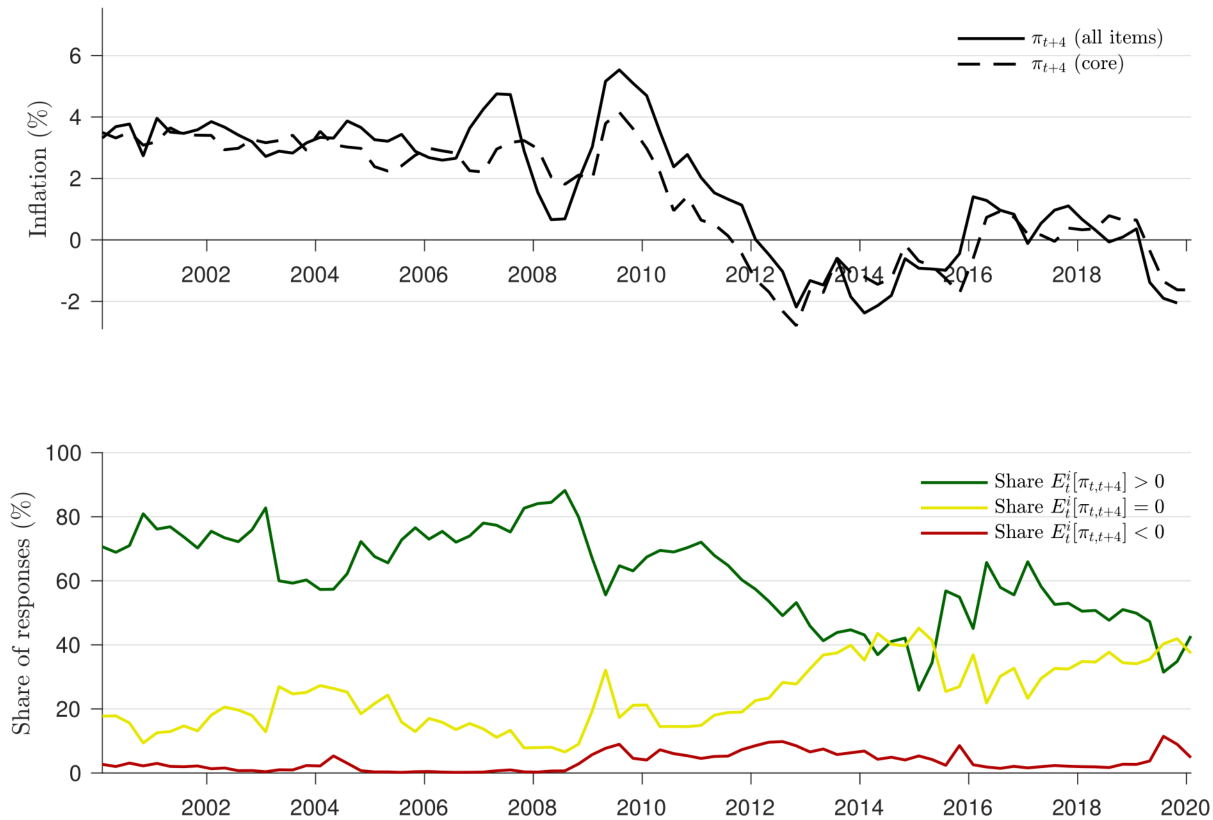

Figure 4: The top panel of the figure shows realized inflation (bold black line) and realized core inflation (dashed black line) in Greece. The bottom panel presents shares of European Commission Business and Consumer Survey participants in Greece who expect positive inflation (green line), zero inflation (yellow line), and negative inflation (red line).

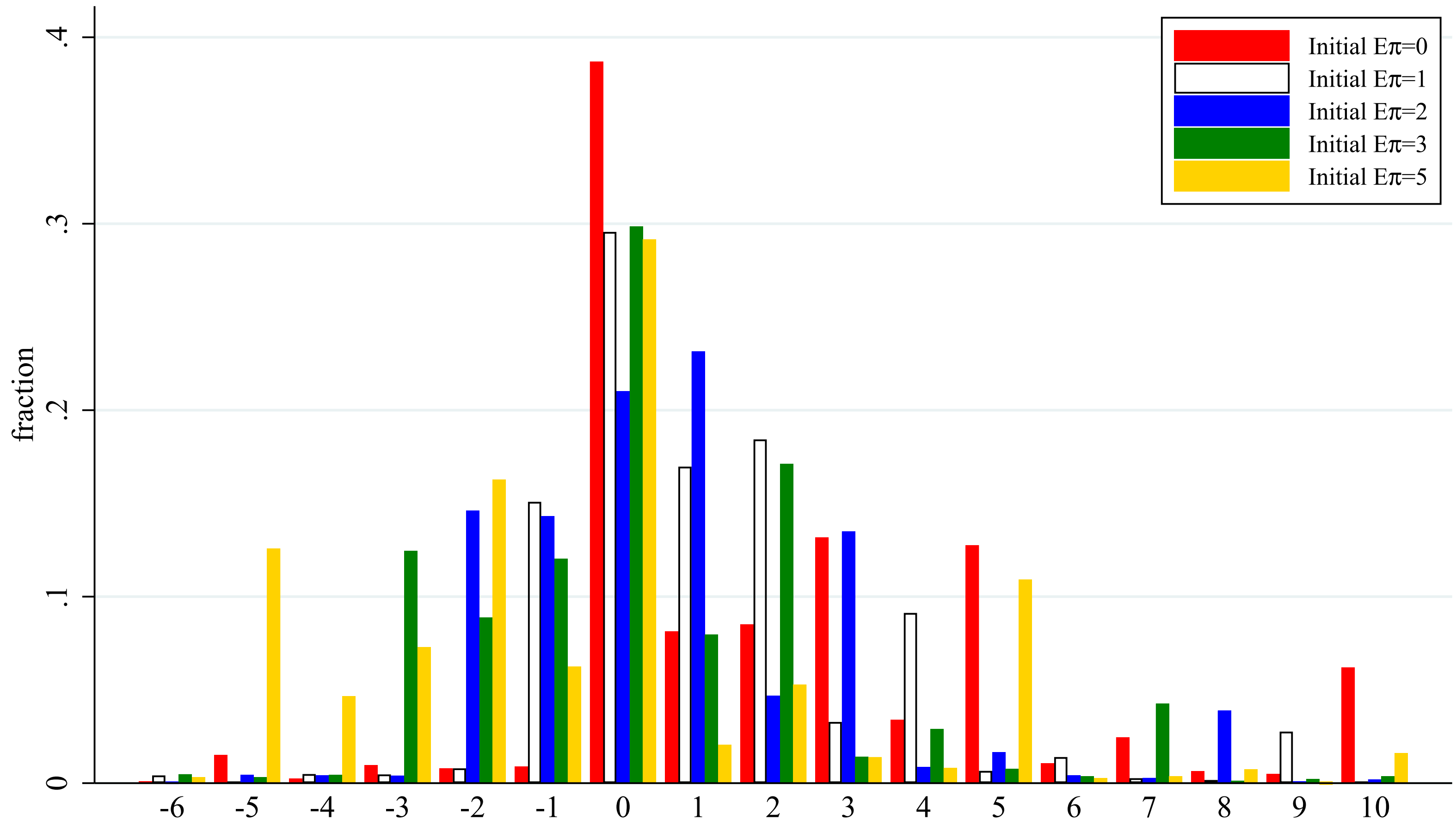

Figure 5: The figure plots five histograms of inflation expectations revisions $\pi^e_{t,t+12,i} - \pi^e_{t-6,t+6,i}$ conditional on five values of the initial inflation expectations: $\pi^e_{t-6,t+6,i} \in \{0, 1, 2, 3, 5\}$, where t is a month subscript and i is the household subscript The horizontal axis is the size of the revision. The data is from the Michigan Survey of Consumers. The sample period is 1978M1-2019M12.

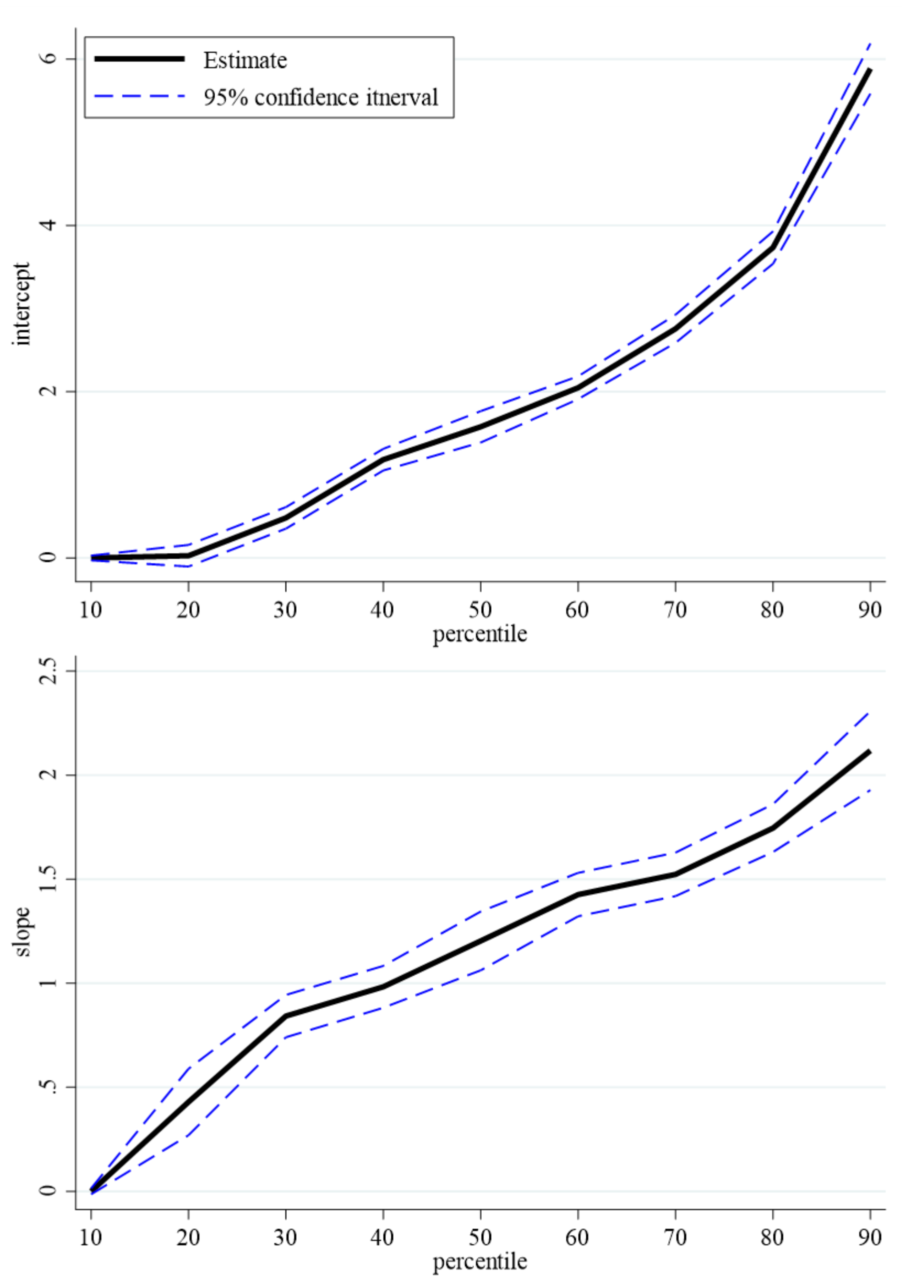

Figure 6: The figure reports estimates of intercept $\alpha^{(\tau)}$ (top panel) and slope $\beta^{(\tau)}$ (bottom panel) estimated for specification 1. The intercept refers to the white, married male respondent who has no children, lives in the West region, and whose highest educational attainment is high school diploma. The solid, thick, black lines are point estimates. The dashed, blue lines are the 95 percent confidence intervals. Confidence intervals are based on standard errors clustered by time. The estimation sample is 1978M1-2019M12.

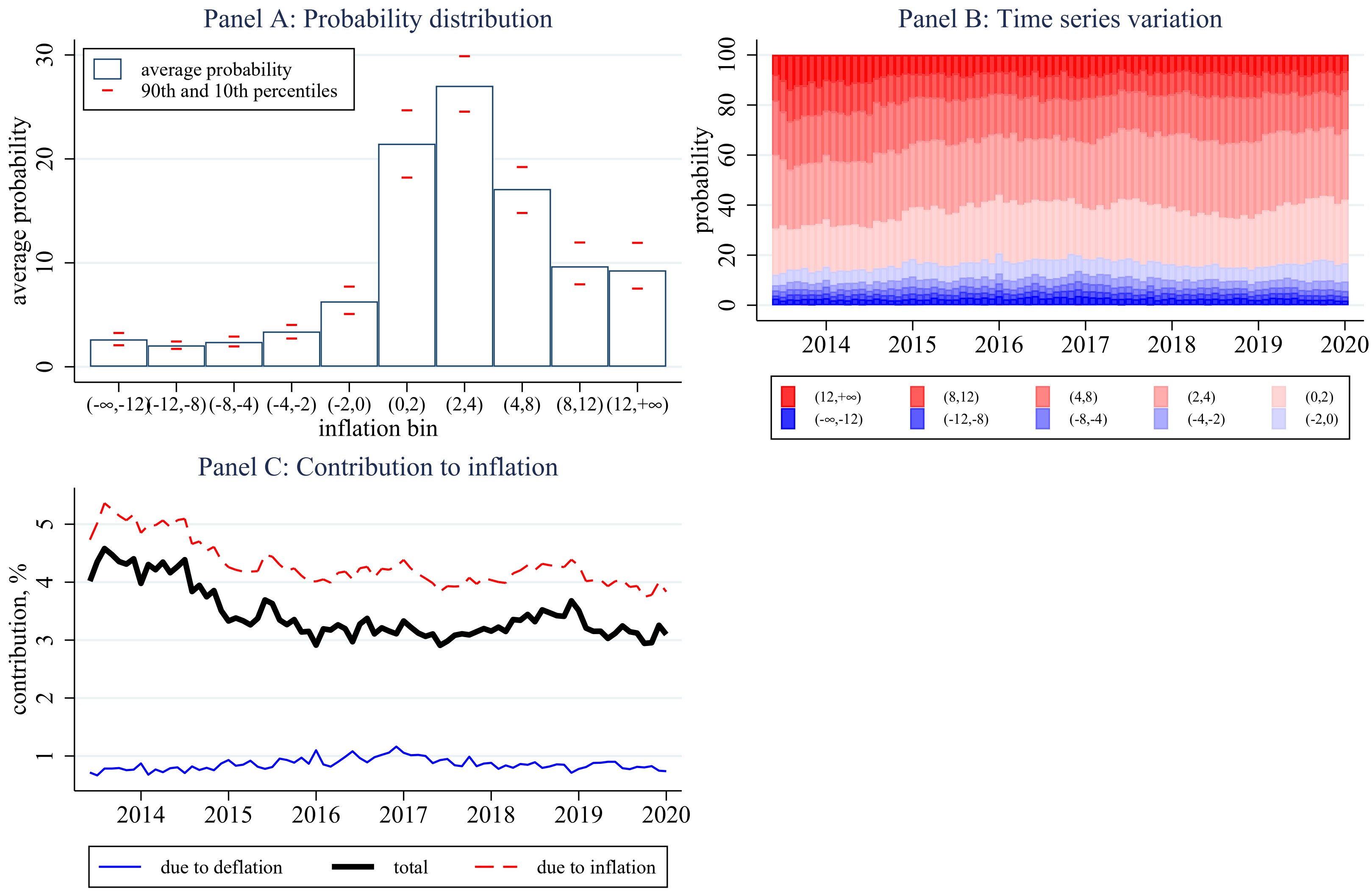

Figure 7: Data are from the Survey of Consumer Expectations. The sample period of all panels is 2013M-2020M1. Panel A is constructed as follows. First, we compute average (across respondents) probability for each bin in a given wave. Second, we compute average (across time) and percentiles. The red whiskers show 90th and 10th percentiles. For Panel B, we compute the average (across respondents) probability assigned to a inflation/deflation bin and plot time series for each inflation/deflation bin. For Panel C, we compute the contribution due to inflation as $\pi^{e,+}_{t,t+12} = 1 \cdot \overline{\mathbb{P}}(\pi_{t,t+12} \in (0,2)) + 2 \cdot \overline{\mathbb{P}}(\pi_{t,t+12} \in (2,4)) + 6 \cdot \overline{\mathbb{P}}(\pi_{t,t+12} \in (4,8)) + 10 \cdot \overline{\mathbb{P}}(\pi_{t,t+12} \in (8,12)) + 14 \cdot \overline{\mathbb{P}}(\pi_{t,t+12} \in (12,\infty))$, where $\overline{\mathbb{P}}(\pi_{t,t+12} \in X)$ is the average (across respondents) probability for inflation being in set X. The contribution due to deflation is $\pi^{e,-}_{t,t+12} = 1 \cdot \overline{\mathbb{P}}(\pi_{t,t+12} \in (-2,0)) + 2 \cdot \overline{\mathbb{P}}(\pi_{t,t+12} \in (-4,-2)) + 6 \cdot \overline{\mathbb{P}}(\pi_{t,t+12} \in (-8,-4)) + 10 \cdot \overline{\mathbb{P}}(\pi_{t,t+12} \in (-12,-8)) + 14 \cdot \overline{\mathbb{P}}(\pi_{t,t+12} \in (-\infty,-12))$; the expected inflation ("total") is computed as $\pi^{e}_{t,t+12} = \pi^{e,+}_{t,t+12} - \pi^{e,-}_{t,t+12}$.

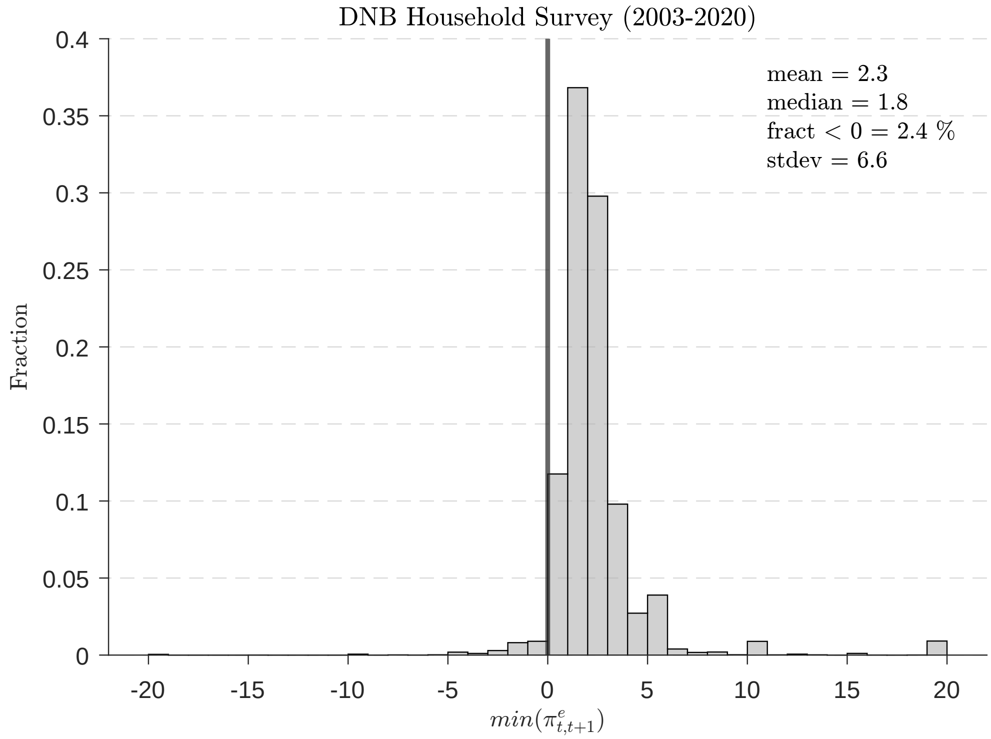

Figure 8: The histogram of one year ahead inflation expectations in Netherlands from the Dutch National Bank Household Survey between 2003 and 2020. The values below -20 percent are set to -20 and above 20 percent are set to 20 on the histogram.

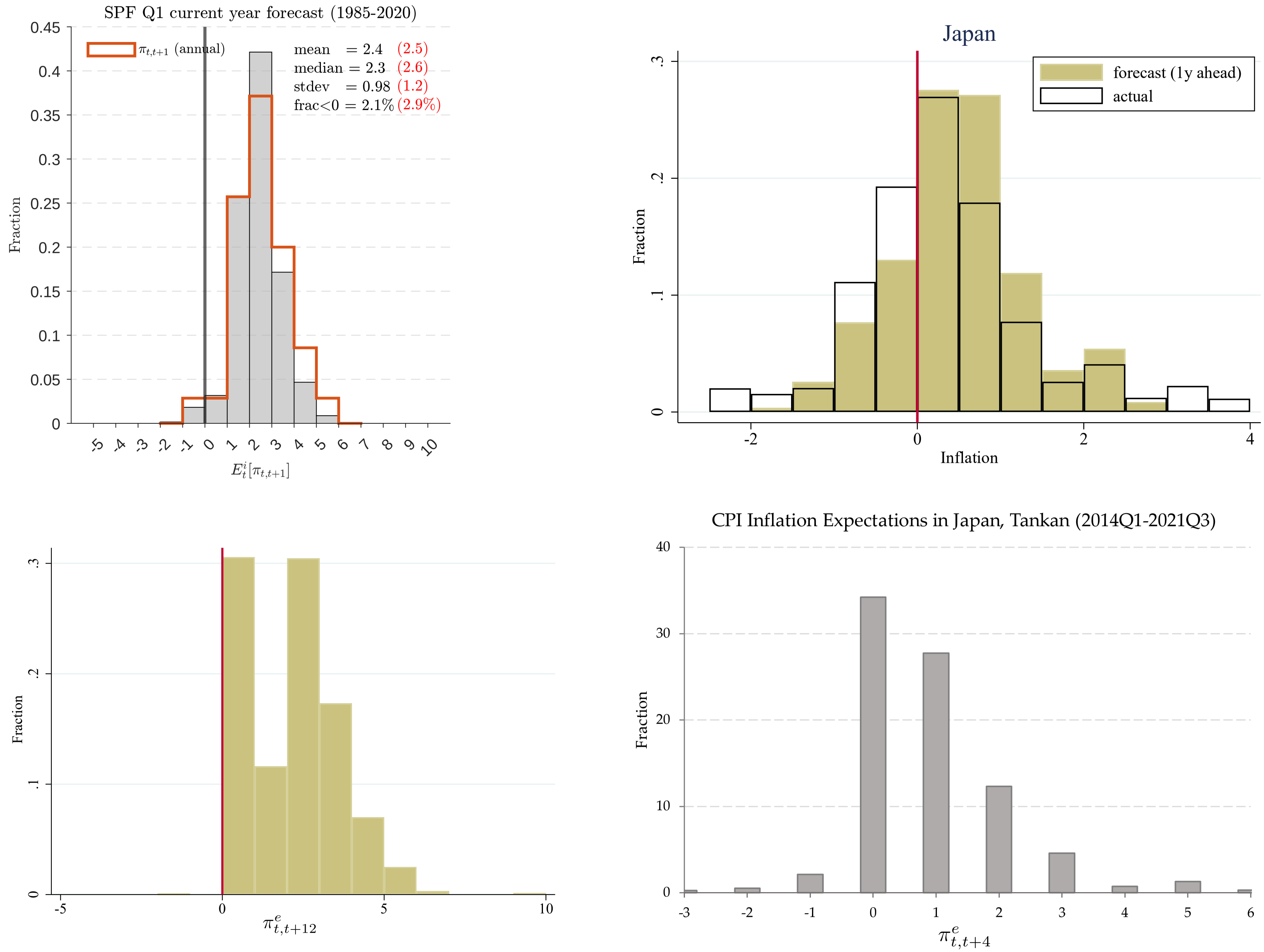

Figure 9: The top left panel is the histogram of one year ahead inflation expectations by professional forecasters in the US from the Survey of Professional Forecasters between 1985 and 2020. The top right panel presents the histogram of one-year ahead inflation expectations by Consensus Economics professional forecasters (olive bars) and realized inflation rate (black lines) in Japan using monthly data from 2005 to 2019. The consensus economics one-year ahead expected inflation is computed as (12-month)*(forecast for the current calendar year)+month*(forecast for the next calendar year). The bottom left panel is the histogram of one year ahead inflation expectations by firm CEOs in the US in 2020Q3 from Candia *et al.* (2021). The bottom right panel is the histogram of one-year ahead consumer price index inflation expectations by enterprises in Japan between 2014Q1 and 2021Q3 from the Short-Term Economic Survey of Enterprises in Japan (TANKAN).

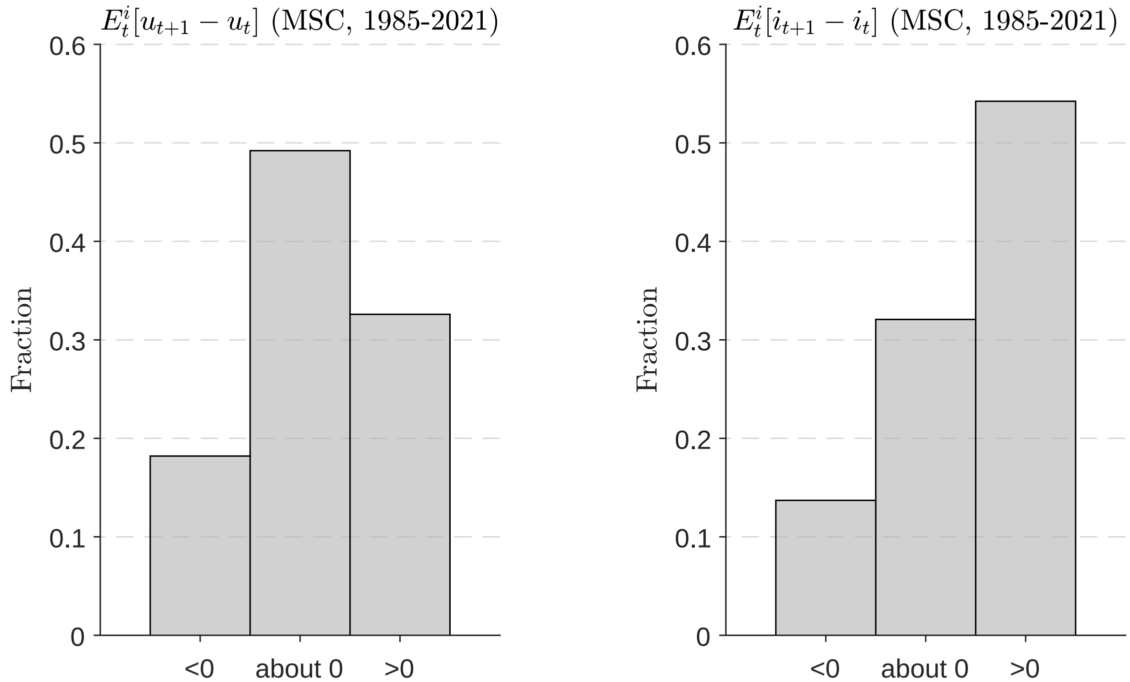

Figure 10: One-year ahead forecast of unemployment changes (the left panel) and interest rate changes (the right panel) from the Michigan Survey of Consumers between 1985 and 2021.

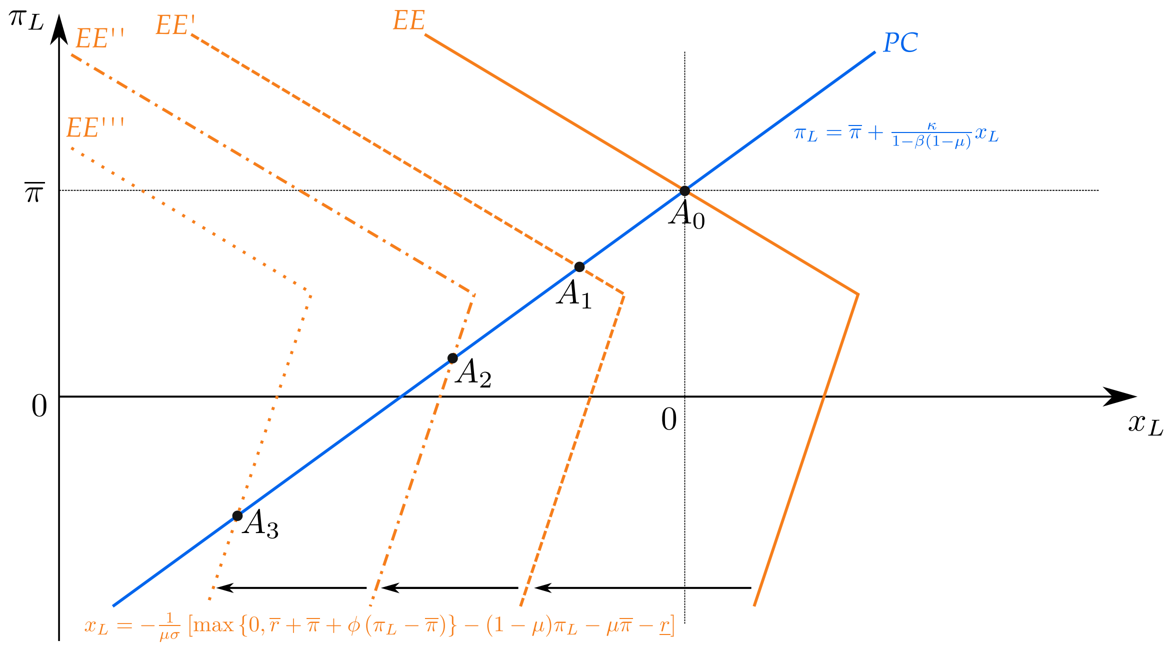

Figure 11: The diagram shows the upward-sloping Phillips curve in solid blue and four piece-wise linear Euler equations. They correspond to $r^n = \overline{r}$ (solid orange line EE), $r^n = \underline{r}$ and $\underline{r}$ is small enough (in absolute value) such that the ELB on the nominal interest rate does not bind in equilibrium (dashed orange line EE'), $r^n = \underline{r}$ and $\underline{r}$ is large enough (in absolute value) such that the ELB on the nominal interest binds in equilibrium but not too large so that the inflation rate is positive in equilibrium (dash-dotted orange line EE''), $r^n = \underline{r}$ and $\underline{r}$ is large enough (in absolute value) such that the ELB on the nominal interest binds and the inflation rate is negative in equilibrium (dotted orange line EE'''), respectively.

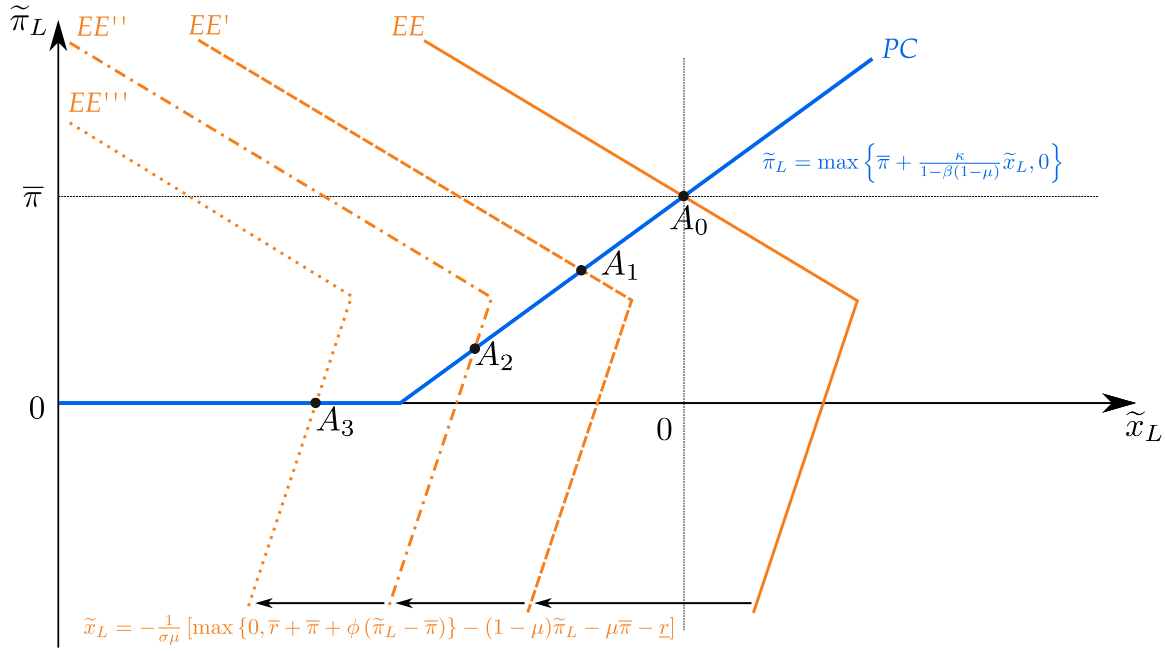

Figure 12: The diagram shows the upward-sloping *forecast* Phillips curve in solid blue and four piece-wise linear *forecast* Euler equations. They correspond to $r^n = \overline{r}$ (solid orange line EE), $r^n = \underline{r}$ and $\underline{r}$ is small enough (in absolute value) such that the ELB on the nominal interest rate does not bind in equilibrium (dashed orange line EE'), $r^n = \underline{r}$ and $\underline{r}$ is large enough (in absolute value) such that the ELB on the nominal interest binds in equilibrium but not too large so that the ZLB on inflation expectations is slack in the intersection A_1 (dash-dotted orange line EE''), $r^n = \underline{r}$ and $\underline{r}$ is large enough (in absolute value) such that the ELB on the nominal interest and the ZLB on inflation expectations bind in the intersection A_3 (dotted orange line EE'''), respectively. The realized inflation rate and output gap values coincide with their forecast values in the case when the Euler equation is in the positions EE, EE', and EE'', but not when the Euler equation is in the position EE'''.

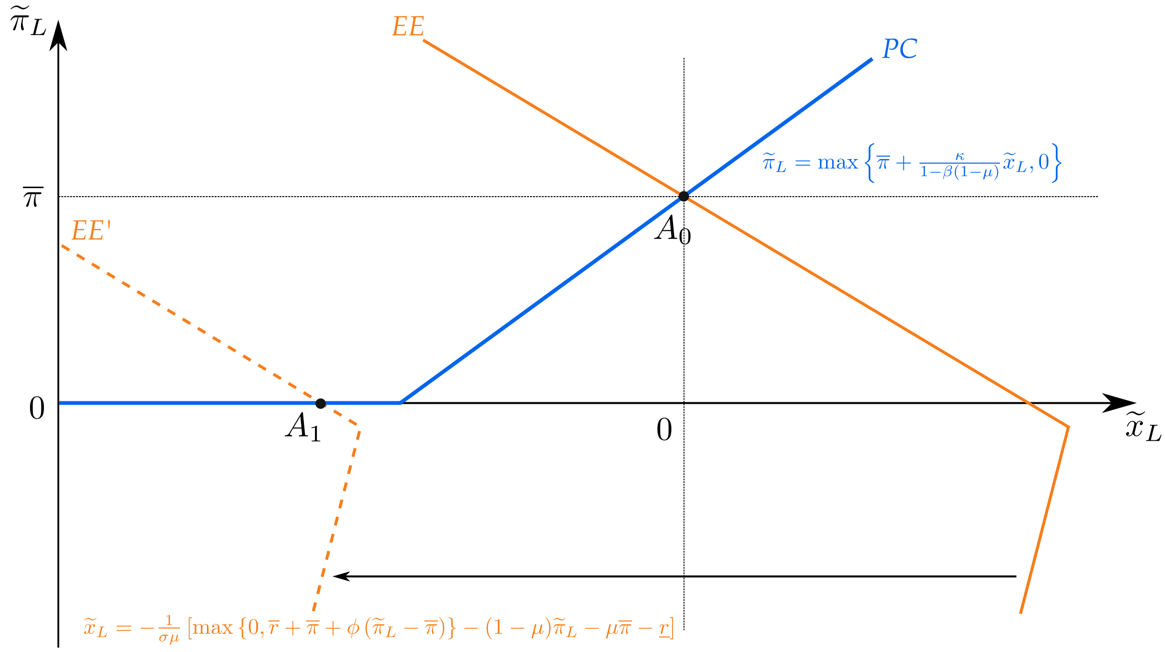

Figure 13: The diagram shows the upward sloping (forecast) Phillips curve PC together with its flat part, which corresponds to the ZLB on inflation forecast and two piece-wise linear (forecast) Euler equations corresponding to $r^n = \overline{r}$ (solid line EE') and $r^n = \underline{r}$ (dashed line EE''), respectively. In the intersection, the ELB on the nominal interest rate does not bind when $r^n = \underline{r}$, however the ZLB on expected inflation does.

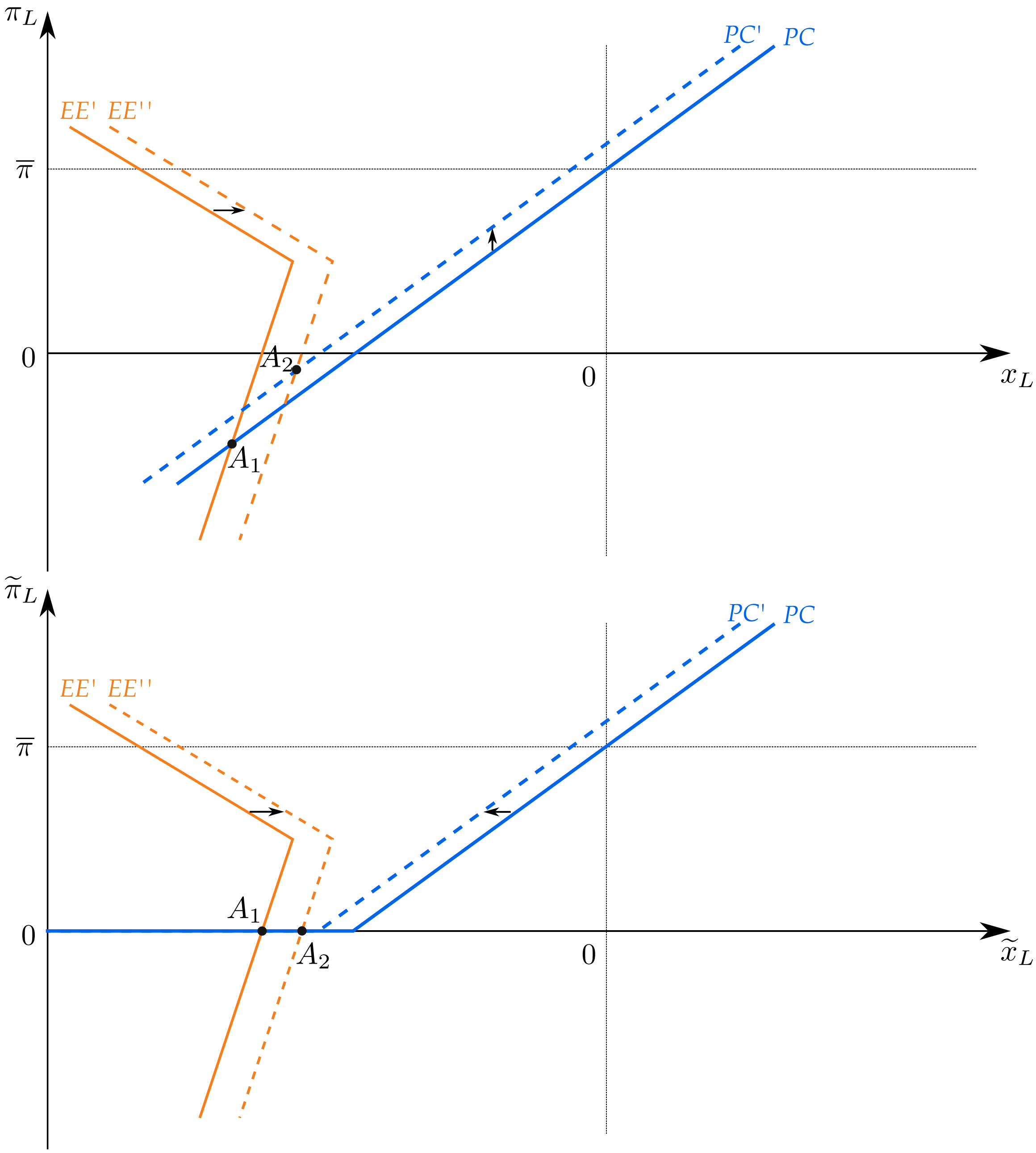

Figure 14: The top panel plots the Euler equation and the Phillips curve when households have FIRE beliefs, while the bottom panel plots expected Euler equation and Phillips curve when beliefs feature the ZLB on inflation expectations. The solid Phillips curves and Euler equations in both panels are plotted without forward guidance, while the dashed Phillips curves and Euler equations correspond to the case with forward guidance.

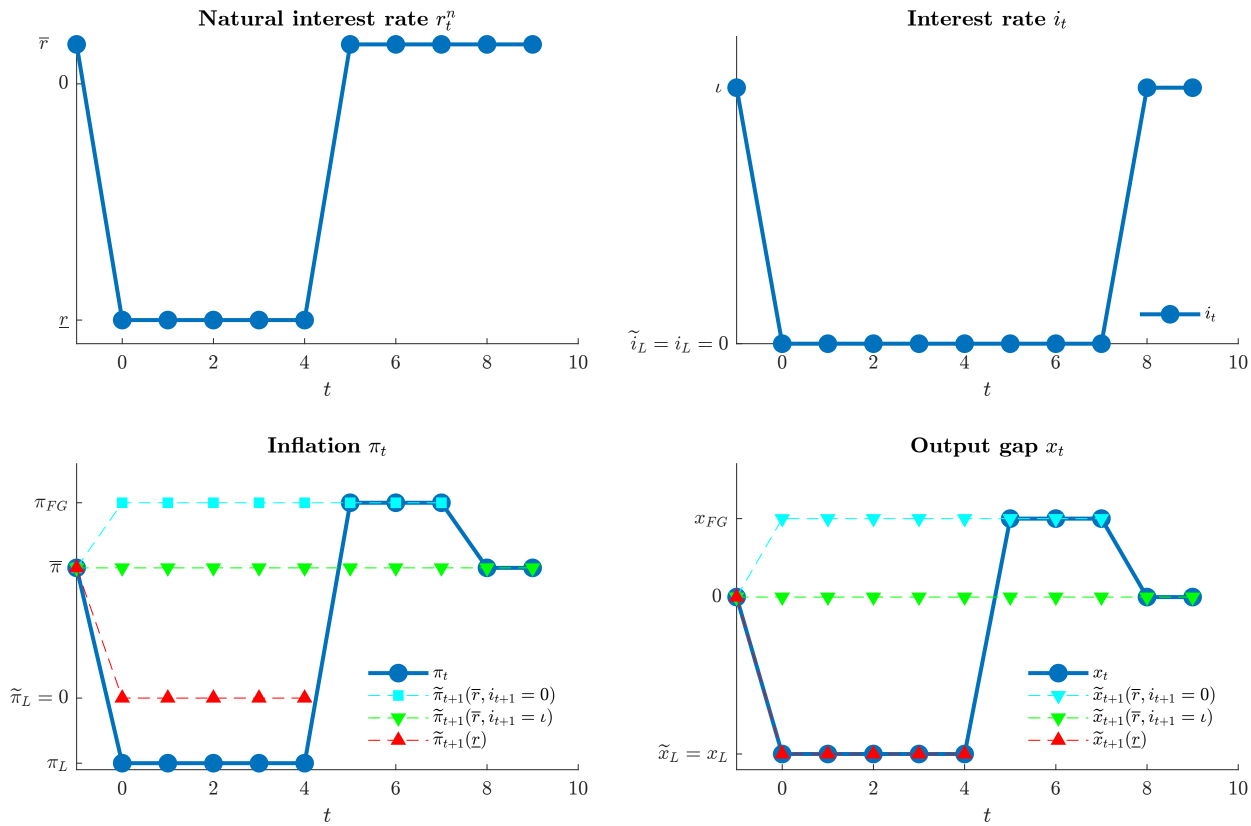

Figure 15: The figure plots the evolution of equilibrium interest rate, output gap, and inflation as well as their forecasts in the possible future states for one particular realization of the path of the natural interest rate that reverts back to its normal level in period $t = 5$ and for one realization of the evolution the low for long monetary policy. The central bank starts following the Taylor rule in period $t = 8$. The ELB on the nominal interest rate binds contemporaneously and in the future under the low realization of the natural interest rate. The expected inflation under the low realization of the natural interest rate is zero.

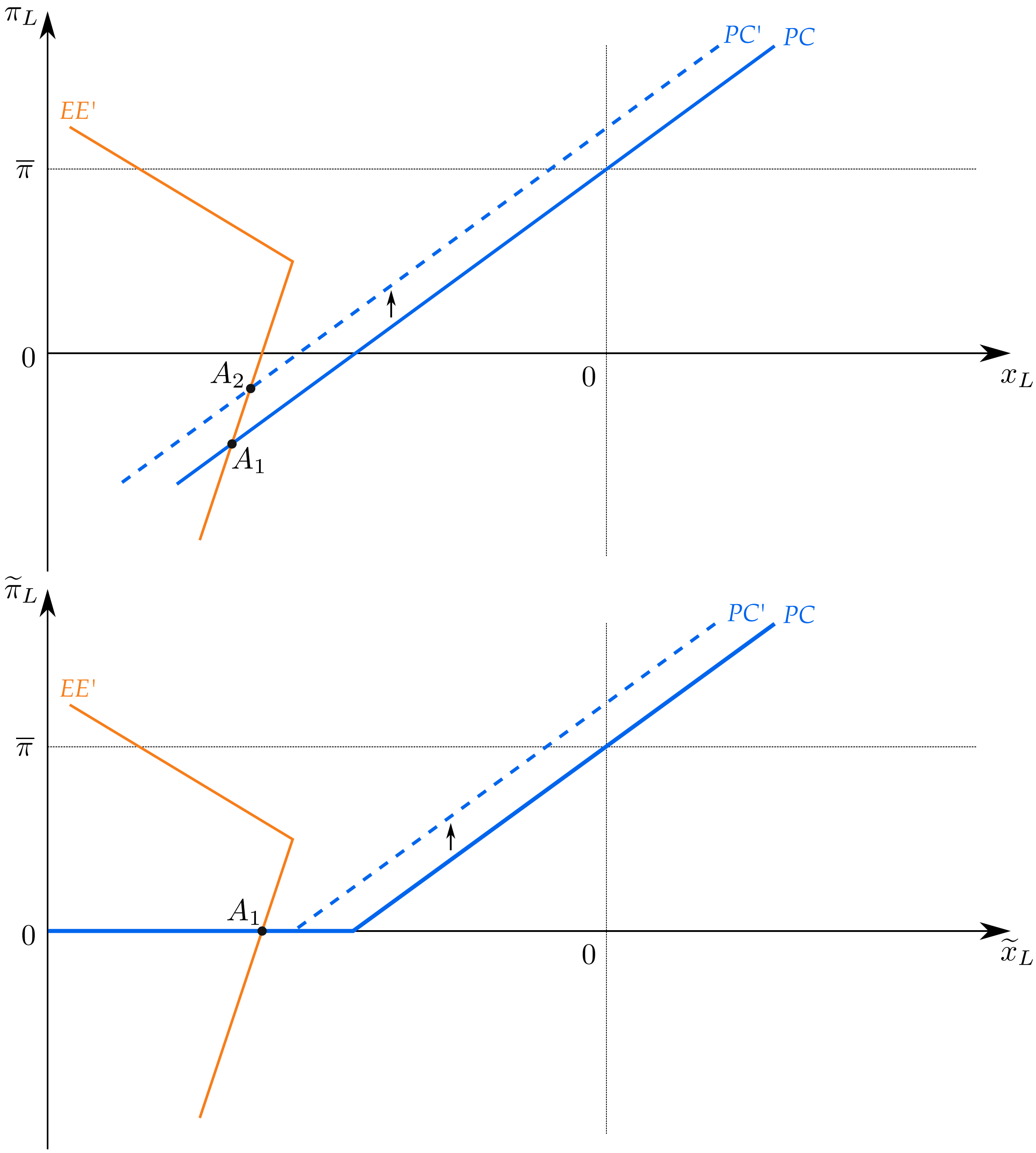

Figure 16: The top panel plots the Euler equation and the Phillips curve when agents have FIRE beliefs, while the bottom panel plots expected Euler equation and Phillips curve when beliefs feature the ZLB on inflation expectations. The solid Phillips curves in both panels show are plotted for the case of $u_t = 0$, while the dashed Phillips curves correspond to $u_t = u > 0$.

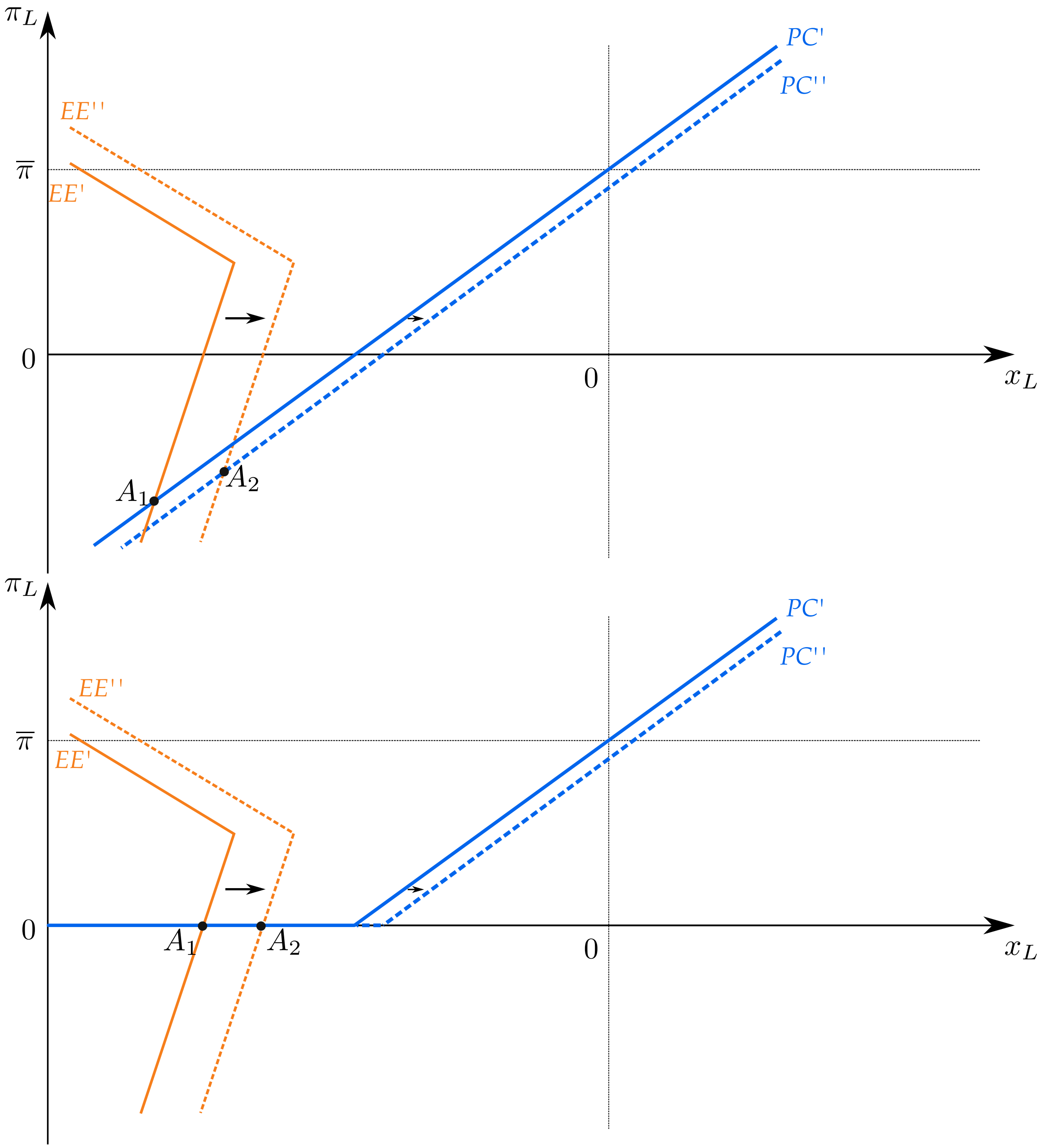

Figure 17: The top panel plots the Euler equation and the Phillips curve when agents have FIRE beliefs, while the bottom panel plots expected Euler equation and Phillips curve when beliefs feature the ZLB on inflation expectations. The dashed curves in both panels are plotted for the case of $g_t = g > 0$, while the solid curves correspond to $g_t = 0$.

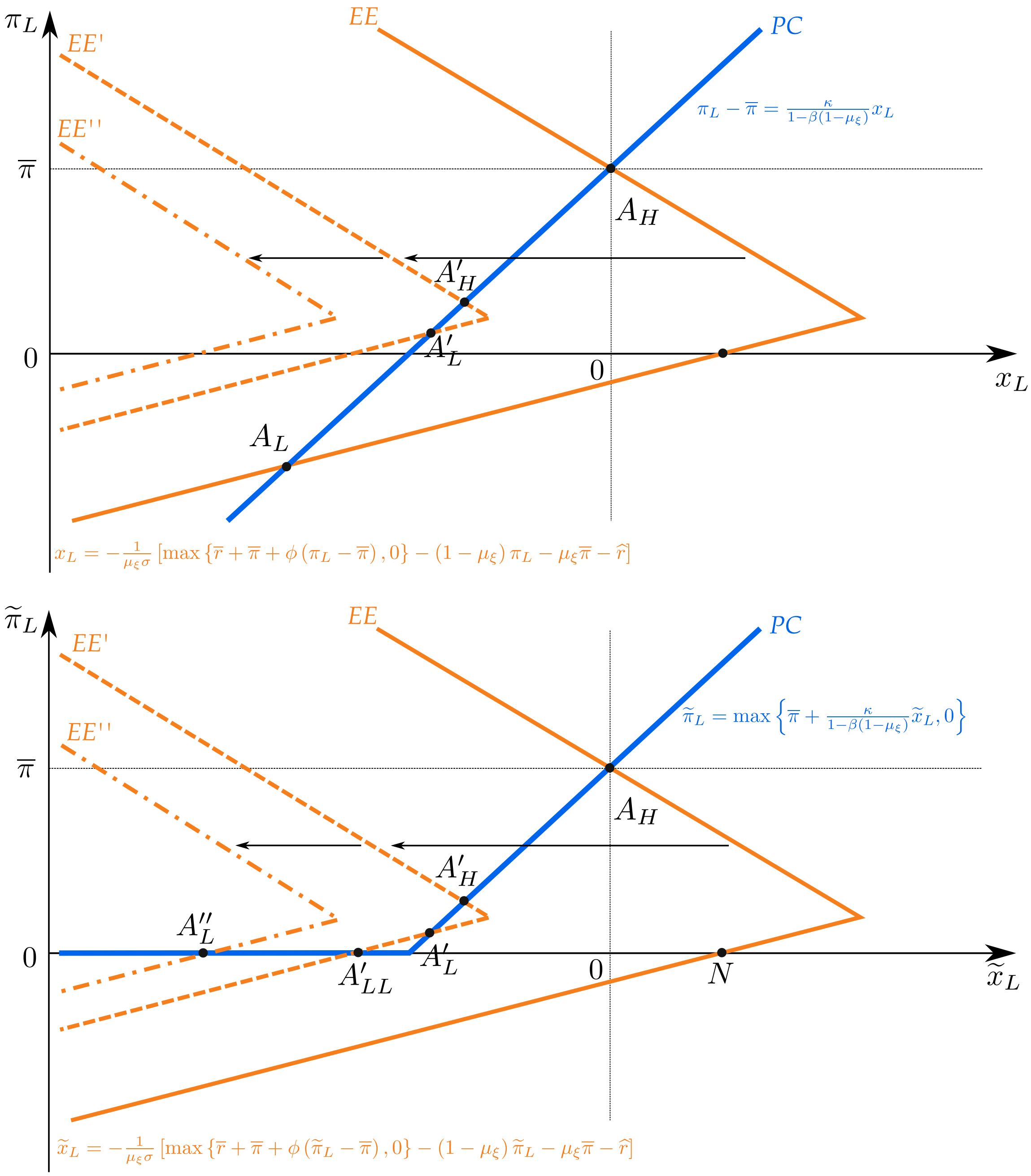

Figure 18: The top panel plots three Euler equation, corresponding to three values of the natural interest rate r'', r', $\overline{r}$, and the Phillips curve when agents have FIRE beliefs, while the bottom panel plots three forecast Euler equations and forecast Phillips curve when beliefs feature the ZLB on inflation expectations.

A Appendix

A.1 The Phillips Curve Derivation

This appendix derives the version of the Phillips curve with the constraint on inflation expectations.

Aggregate Price Level. The contemporaneous aggregate price level is

$$\begin{aligned} P_t &= \left[\int \left[\Pi P_{t-1}(i) \right]^{1-\epsilon} di + (1-\theta) \left(P_t^* \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \\ &= \left[\theta P_{t-1}^{1-\epsilon} \overline{\Pi}^{1-\epsilon} + (1-\theta) \left(P_t^* \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}, \end{aligned}$$

where P_t^* is the optimal price and $1-\theta$ is the fraction of firms that change their prices in period t. Note that we took account that those firms that do not set their optimal prices nevertheless change their prices by the steady state level of inflation due to indexation. The contemporaneous inflation rate is

$$\Pi_t^{1-\epsilon} = \left(\frac{P_t}{P_{t-1}} \right)^{1-\epsilon} = \theta \overline{\Pi}^{1-\epsilon} + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon}.$$

In the steady state with gross inflation $\overline{\Pi}$, we have $P_t = P_t^*$ and $P_t/P_{t-1} = P_t^*/P_{t-1} = \overline{\Pi}$. Log-linearzing around $\Pi_t = \overline{\Pi}$, we get

$$\pi_t - \overline{\pi} = (1-\theta) \left(p_t^* - p_{t-1} - \overline{\pi} \right),$$

where $\pi_t \equiv \log \left(P_t / P_{t-1} \right)$, $\overline{\pi} \equiv \log \overline{\Pi}$, $p_t^* = \log P_t^*$, and $p_t = \log P_t$.

Note that the same equation must hold for future forecasts of the aggregate price and inflation. Specifically,

$$\widetilde{\pi}_{t+1} - \overline{\pi} = (1-\theta) \left(\widetilde{p}_{t+1}^* - p_t - \overline{\pi} \right),$$

and

$$\widetilde{\pi}_{t+s} - \overline{\pi} = (1-\theta) \left(\widetilde{p}_{t+s}^* - \widetilde{p}_{t+s-1} - \overline{\pi} \right),$$

when $s > 1$. Also note that because expected inflation cannot be negative,

$$\widetilde{\pi}_{t+1} = \max \left\{ \pi + (1-\theta) \left(\widetilde{p}_{t+1}^* - p_t - \overline{\pi} \right), 0 \right\}, \tag{A.1}$$

$$\widetilde{\pi}_{t+s} = \max \left\{ \pi + (1-\theta) \left(\widetilde{p}_{t+s}^* - \widetilde{p}_{t+s-1} - \overline{\pi} \right), 0 \right\}, \tag{A.2}$$

for $s > 1$.

Reset prices. Firm i operates the following production function

$$Y_{i,t} = A_t N_{i,t}^{1-\alpha}.$$

This relationship holds contemporaneously and it is also expected to hold in the future.

The household labor supply optimality condition is

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi},$$

which is derived for the households whose utility function is $U = C_t^{1-\sigma}/(1-\sigma) - N_t^{1+\varphi}/(1+\varphi)$.

The demand that firm i expects to face in period $t+s$ when it updated its price in period t is

$$\widetilde{Y}_{i,t+s}(P_t^*) = \left(\frac{\overline{\Pi}^s P_t^*}{\widetilde{P}_{t+s}}\right)^{-\epsilon} \widetilde{C}_{t+s}$$

As a result, the forecast of the *real* labor costs in period $t+s$ of the firm that produces $\widetilde{Y}_{i,t+s}(P_t^*)$ is

$$\begin{aligned}
\widetilde{\Psi}_{t+s}(\widetilde{Y}_{i,t+s}(P_t^*)) &= \frac{\widetilde{W}_{t+s}}{\widetilde{P}_{t+s}} \widetilde{N}_{t+s} \\
&= \frac{\widetilde{W}_{t+s}}{\widetilde{P}_{t+s}} \left(\frac{\widetilde{Y}_{i,t+s}(P_t^*)}{A_t}\right)^{\frac{1}{1-\alpha}} \\
&= \widetilde{C}_{t+s}^{\sigma} \widetilde{N}_{t+s}^{\varphi} \left(\frac{\widetilde{Y}_{i,t+s}(P_t^*)}{\widetilde{A}_{t+s}}\right)^{\frac{1}{1-\alpha}} \\
&= \widetilde{Y}_{t+s}^{\sigma+\frac{\varphi}{1-\alpha}} \widetilde{A}_{t+s}^{-\frac{1+\varphi}{1-\alpha}} \left[\widetilde{Y}_{i,t+s}(P_t^*)\right]^{\frac{1}{1-\alpha}}.
\end{aligned}$$

The firm that resets its price in period t solves

$$\max_{P_t^*} \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left[Q_{t,t+s} \left\{P_t^* \overline{\Pi}^s \widetilde{Y}_{i,t+s}(P_t^*) - \widetilde{P}_{t+s} \widetilde{\Psi}_{t+s}(\widetilde{Y}_{i,t+s}(P_t^*))\right\}\right],$$

where

$$Q_{t,t+s} = \beta^s \left(\frac{\widetilde{C}_{t+s}}{C_t}\right)^{-\sigma} \frac{P_t}{\widetilde{P}_{t+s}}$$

Optimal choice of P_t^* yields

$$0 = \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left[Q_{t,t+s} \widetilde{Y}_{i,t+s}(P_t^*) \left\{P_t^* \overline{\Pi}^s - \frac{\epsilon}{\epsilon-1} \widetilde{P}_{t+s} \widetilde{\Psi}'_{t+s}(\widetilde{Y}_{i,t+s}(P_t^*))\right\}\right].$$

Use the fact that $\widetilde{\Psi}'_{t+s}(\widetilde{Y}_{i,t+s}(P_t^*)) = \widetilde{\Psi}_{t+s}(\widetilde{Y}_{i,t+s}(\widetilde{P}_{t+s})) \left(\overline{\Pi}^s P_t^* / \widetilde{P}_{t+s}\right)^{\epsilon\alpha/(1-\alpha)}$

$$0 = \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left[Q_{t,t+s} \widetilde{Y}_{i,t+s}(P_t^*) \left\{(P_t^*)^{1-\epsilon\alpha/(1-\alpha)} \overline{\Pi}^s - \frac{\epsilon}{\epsilon-1} \widetilde{P}_{t+s}^{1-\epsilon\alpha/(1-\alpha)} \widetilde{\Psi}'_{t+s}(\widetilde{Y}_{i,t+s}(\widetilde{P}_{t+s})) \left(\overline{\Pi}^s\right)^{\epsilon\alpha/(1-\alpha)}\right\}\right].$$

Divide by $(P_{t-1})^{1-\epsilon\alpha/(1-\alpha)}$

$$0 = \sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left[Q_{t,t+s} \widetilde{Y}_{i,t+s}(P_t^*) \left\{\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon\alpha/(1-\alpha)} \overline{\Pi}^s - \frac{\epsilon}{\epsilon-1} \left(\frac{\widetilde{P}_{t+s}}{P_{t-1}}\right)^{1-\epsilon\alpha/(1-\alpha)} \widetilde{\Psi}'_{t+s}(\widetilde{Y}_{i,t+s}(\widetilde{P}_{t+s})) \left(\overline{\Pi}^s\right)^{\epsilon\alpha/(1-\alpha)}\right\}\right].$$

Log-linearize around the steady state with inflation Π where $P_t^*/P_{t-1} = \overline{\Pi}$, $P_t^* = P_{t+s}/\overline{\Pi}^s$, $Y_t = C_t = C = Y$, $Q_{t,t+s} = \beta^s/\overline{\Pi}^s$, $\Psi'_{t+s}(Y_{i,t+s}(P_{t+s})) = (\epsilon-1)/\epsilon$ (including expectations).

$$p_t^* - p_{t-1} - \overline{\pi} = (1-\beta\theta) \sum_{s=0}^{\infty} (\beta\theta)^s \mathbb{E}_t \left[\Omega \widehat{\widetilde{mc}}_{t+s} + \widetilde{p}_{t+s} - p_{t-1} - (s+1)\overline{\pi}\right]$$

where

$$\Omega \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon},$$

and

$$mc_{t+s} \equiv \Psi'_{t+s}(Y_{i,t+s}(P_{t+s})),$$

while $\widehat{mc}_{t+s}$ is the log deviation of the marginal cost from its steady state value.

Rewrite it a bit

$$\begin{aligned}
p_t^* - p_{t-1} - \overline{\pi} &= (1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^s\,\mathbb{E}_t\left[\Omega\widehat{mc}_{t+s} + \sum_{k=0}^{s}(\widetilde{\pi}_{t+k}-\overline{\pi})\right] \\
&= (1-\beta\theta)\,\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\theta)^s\,\Omega\widehat{mc}_{t+s}\right] + (1-\beta\theta)\sum_{s=0}^{\infty}\sum_{k=0}^{s}(\beta\theta)^s\,\mathbb{E}_t\,(\widetilde{\pi}_{t+k}-\overline{\pi}) \\
&= (1-\beta\theta)\,\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\theta)^s\,\Omega\widehat{mc}_{t+s}\right] + \sum_{s=0}^{\infty}(\beta\theta)^s\,\mathbb{E}_t\left[\widetilde{\pi}_{t+s}-\overline{\pi}\right].
\end{aligned}$$

By repeating the same steps, we can derive the forecast versions of the last equation

$$\widetilde{p}_{t+1}^* - p_t - \overline{\pi} = (1-\beta\theta)\,\mathbb{E}_{t+1}\left[\sum_{s=0}^{\infty}(\beta\theta)^s\,\Omega\widehat{mc}_{t+1+s}\right] + \sum_{s=0}^{\infty}(\beta\theta)^s\,\mathbb{E}_{t+1}\left[\widetilde{\pi}_{t+1+s}-\overline{\pi}\right], \tag{A.3}$$

and

$$p_{t+s}^* - p_{t+s-1} - \overline{\pi} = (1-\beta\theta)\,\mathbb{E}_{t+s}\left[\sum_{k=0}^{\infty}(\beta\theta)^k\,\Omega\widehat{mc}_{t+s+k}\right] + \sum_{k=0}^{\infty}(\beta\theta)^k\,\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+k}-\overline{\pi}\right], \tag{A.4}$$

for $s > 1$. Combining equations (A.1) and (A.2) together with (A.3) and (A.4), we can write

$$\widetilde{\pi}_{t+s} = \max\left\{\overline{\pi} + (1-\theta)\left((1-\beta\theta)\,\mathbb{E}_{t+s}\left[\sum_{k=0}^{\infty}(\beta\theta)^k\,\Omega\widehat{mc}_{t+s+k}\right] + \sum_{k=0}^{\infty}(\beta\theta)^k\,\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+k}-\overline{\pi}\right]\right), 0\right\},$$

for $s \geq 1$. Because $\widetilde{\pi}_{t+s}$ is present both on the left- and the right-hand sides of the last equation, we simplify it as

$$\widetilde{\pi}_{t+s} = \max\left\{\overline{\pi} + \frac{1-\theta}{\theta}\left((1-\beta\theta)\,\mathbb{E}_{t+s}\left[\sum_{k=0}^{\infty}(\beta\theta)^k\,\Omega\widehat{mc}_{t+s+k}\right] + \beta\theta\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1+k}-\overline{\pi}\right]\right), 0\right\}.$$

Finally, the contemporaneous Phillips curve is

$$\pi_t = \overline{\pi} + \frac{1-\theta}{\theta}\left\{(1-\beta\theta)\,\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k\,\Omega\widehat{mc}_{t+k}\right] + \beta\theta\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathbb{E}_t\left[\widetilde{\pi}_{t+1+k}-\overline{\pi}\right]\right\}.$$

Does the last equation reduce to the standard Phillips curve when the on ZLB on IE does not bind? Start with the future Phillips curve

$$\widetilde{\pi}_{t+s} - \overline{\pi} = \frac{(1-\theta)(1-\beta\theta)\Omega}{\theta}\sum_{k=0}^{\infty}(\beta\theta)^k\mathbb{E}_{t+s}\left[\widehat{mc}_{t+s+k}\right] + \frac{1-\theta}{\theta}\sum_{k=0}^{\infty}(\beta\theta)^{k+1}\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1+k}-\overline{\pi}\right].$$

Make the following transformations

$$
\begin{aligned}
\frac{\widetilde{\pi}_{t+s}-\overline{\pi}}{1-\theta} =&\frac{(1-\beta\theta)\Omega}{\theta}\sum_{k=0}^{\infty}(\beta\theta)^{k}\mathbb{E}_{t+s}\left[\widehat{\widehat{mc}}_{t+s+k}\right]+\frac{1}{\theta}\sum_{k=0}^{\infty}(\beta\theta)^{k+1}\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1+k}-\overline{\pi}\right] \\
=&\frac{(1-\beta\theta)\Omega}{\theta}\widehat{\widehat{mc}}_{t+s}+\beta\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1}-\overline{\pi}\right]+\frac{(1-\beta\theta)\Omega}{\theta}\sum_{k=1}^{\infty}(\beta\theta)^{k}\mathbb{E}_{t+s}\left[\widehat{\widehat{mc}}_{t+s+k}\right] \\
&+\frac{1}{\theta}\sum_{k=1}^{\infty}(\beta\theta)^{k+1}\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1+k}-\overline{\pi}\right] \\
=&\frac{(1-\beta\theta)\Omega}{\theta}\widehat{\widehat{mc}}_{t+s}+\beta\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1}-\overline{\pi}\right]+\beta\theta \\
&\left\{\frac{(1-\beta\theta)\Omega}{\theta}\sum_{k=0}^{\infty}(\beta\theta)^{k}\mathbb{E}_{t+s}\left[\widehat{\widehat{mc}}_{t+s+k+1}\right]+\frac{1}{\theta}\sum_{k=0}^{\infty}(\beta\theta)^{k+1}\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+2+k}-\overline{\pi}\right]\right\} \\
=&\frac{(1-\beta\theta)\Omega}{\theta}\widehat{\widehat{mc}}_{t+s}+\beta\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1}-\overline{\pi}\right]+\beta\theta\mathbb{E}_{t+s}\left\{\frac{\widetilde{\pi}_{t+s+1}}{1-\theta}\right\},
\end{aligned}
$$

and arrive at

$$
\widetilde{\pi}_{t+s}-\overline{\pi}=\frac{(1-\theta)(1-\beta\theta)\Omega}{\theta}\widehat{\widehat{mc}}_{t+s}+\beta\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1}-\overline{\pi}\right],
$$

which is the standard New Keynesian Phillips curve with non-zero inflation target.

Replacing marginal costs with output gap. We use the equation

$$
\widehat{mc}_t=\left(\sigma+\frac{\varphi+\alpha}{1-\alpha}\right)x_t.
$$

$$
\widetilde{\pi}_{t+s}-\overline{\pi}=\max\left\{\kappa\sum_{k=0}^{\infty}(\beta\theta)^{k}\mathbb{E}_{t+s}\left[x_{t+s+k}\right]+\frac{1-\theta}{\theta}\sum_{k=0}^{\infty}(\beta\theta)^{k+1}\mathbb{E}_{t+s}\left[\widetilde{\pi}_{t+s+1+k}-\overline{\pi}\right],0\right\},
$$

where

$$
\kappa\equiv\frac{(1-\theta)(1-\beta\theta)\Omega}{\theta}\left(\sigma+\frac{\varphi+\alpha}{1-\alpha}\right).
$$